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## Gas Storage and Security of Supply in the Medium Run

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*Abstract*

This paper analyzes the role of storage for economies facing the risk of a gas supply disruption. We characterize the optimal/competitive transitory dynamics (accumulation, drainage and target stock). We partially relax the irreversibility hypothesis, by extending the model in two directions: first, we consider a long but finite duration of the crisis, and second, we study the impact of "alerts". The policy analysis shows that the lack of protection of property rights, e.g. antispeculation measures, is likely to discourage storage completely. Responsible policy involves a series of measures taken ex ante that limit market failure. We provide a method to calculate the social value of a policy. Finally, the model is extended to encompass specific characteristics of the gas industry (injection and release costs, limited storage capacity).

*Keywords:* Security of supply, Gas Industry
*JEL classification:* L95, L51, C61

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# Gas Storage and Security of Supply in the Medium Run*

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# 1 Introduction

## 1.1 Supply disruption and storage

As security of gas supply raises serious concerns, strategies against disruption are becoming of crucial importance in Europe. In 2003, about one quarter of the EU primary energy consumption was based on natural gas, and imports from neighboring producers, mainly Russia, accounted for 46% of the total EU15 demand (Eurogas, 2004). Dependency on external supplies is going to increase in the next years, as gas consumption in Europe is expected to grow whereas indigenous sources are forecasted to slow down. Including the new member countries, the European dependence rate for gas will amount to 50% in 2010, 62% in 2020 and 70% in 2030 (Commission of the European Communities, 2002).

By diversifying the risk of disruption and financing pipeline construction, long-term contracts with producers are the primary supply instruments. Security of supply targets can also be met by increasing system flexibility (fuel switching, interruptible contracts, cross-border pipeline capacity and liquid spot markets). However, these mechanisms have a limited capacity to absorb

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shocks that would endanger all the European countries at the same time (accident, civil war or terrorist attack). To ensure uninterrupted services in the short-medium term, precautionary gas storage is indispensable.

The conditions to be fulfilled in relation to security of supply and availability of storage for existing suppliers and entrants have been specified by national laws in application of the Directive 98/30/EC on the liberalization of the gas market.[1] These rules are now potentially subject to change, as European discipline has continued to stress the matter of security of supply both in the new Directive 2003/55/EC, fostering competition in gas markets, and in Directive 2004/67/EC. The latter obliges European countries—by May 2006—to define the roles and responsibilities of all market players in ensuring gas availability and set minimum targets for gas storage, at national or industry level. The storage policy has to be transparent, and member states have to publish regular reports on emergency mechanisms and the levels of gas in storage that the Commission will monitor—a procedure which to date is in place in the US only.

The issue is a very complex one, so simplification is essential if any progress is to be made. We assume in most of the paper that the size of disruption is single-valued and known, its probability is also known and stationary, and disruption marks a permanent transition to a state of lower excess supply. Given these assumptions, we derive the dynamics of accumulation and drawdown in a continuous time context.

Section 2 presents the model. Private stockholding decisions balance the valorization of gas in the event of a crisis, with the carrying costs (capital immobilization and technical costs). We make no distinction between domestic and foreign production in this analysis. Moreover, we focus on the medium term in which both the seasonality of demand (short term) and the exhaustibility of gas (long term) can be practically neglected.[2]

In Section 3, we characterize the competitive equilibrium. Stockpiling before the disruption increases gas prices, so accumulation is all the faster in so far as potential profits loom large. The limiting factor to accumulation is that the value of the stored cubic meter in case of crisis decreases as stocks

[1] In Italy, entrants importing non-EU gas are required to hold stocks equivalent to 10% of the annual supply. In Spain, overall gas supply dependence upon any single external supply source must not exceed 60% and gas companies are obliged to keep gas reserves of at least 35 days of supply. In Denmark, the integrated gas firm has designed its back-up and storage capacity to be able to continue supplies to the non-interruptible market in case of a disruption of one of the two offshore pipelines supplying gas to the country. In France, strategic stocks can withstand disruption of the largest source of supply up to one year.

[2] See Chaton, Creti and Villeneuve (2005) for a complementary approach.

pile up. As the growth of precautionary storage progressively slows down, there is a target stock that will never be exceeded. The comparative statics gives scenarios for possible substitutions between precautionary stocks and transport infrastructures.

The irreversibility hypothesis essentially allows to solve the model by backward induction and is less restrictive than it would appear to be. The dynamics described is exact for any situation in which the utilization of precautionary stocks is shorter than the time required to find alternative supplies. Reasonable parameters (interest rate, storage costs, crisis probability, extent of the crisis) support this approach. Moreover, partially relaxing the irreversibility hypothesis, we study the implications of forewarning of a crisis for storage behavior (Section 4).

If crises were not irreversible, the model dynamics would be far more complex. In a companion paper, Creti and Villeneuve (2006) develop an algorithm for solving a Markovian version of the model in which crises are of variable durations. Though this latter approach may be deemed more realistic, its drawback is that most results are based on simulations. On the contrary, the computational ease due to the irreversibility hypothesis enables us to derive explicit solutions for equilibrium prices, stocks and drainage time. Most importantly, we provide a complete theoretical treatment of the effects of public interventions (Section 5), which is, ultimately, the main focus of this paper. The understanding of potential market failures or imperfections is of crucial importance in the perspective of the European Directive aimed at improving the security of gas supply. For example, stockholders may fear antispeculation measures taken once the crisis has occurred. We show that this lack of protection of property rights is likely to discourage storage completely, and that responsible policy consists in a series of measures (subsidies, public agency) taken ex ante.

We provide in Section 6 a method to evaluate storage policies in a dynamic setting and apply it to simulate the relative cost of imperfect policies in a detailed example.

In the last part of the paper, we suggest two important extensions of the basic model that deal with specific characteristics of the gas industry: non negligible injection and release costs, and limited storage capacity (Section 7).

## 1.2 Related literature

The theoretical literature on energy supply security has mostly been inspired by the question of oil. The decision of the United States to develop strategic petroleum reserves in the aftermath of the OPEC embargo during the '70s motivated studies into the role of stockpiling as a precautionary reserve. There are two sets of models, mostly inspired by the theory of exhaustible resources: works that consider the extraction rate of one country when foreign import, though needed to complement national production, can suddenly default, and those that introduce strategic behavior of consuming countries confronting oligopolistic or cartelized supply.

The first group of models shows that generally, with imports subject to disruption, an importing country faces a trade-off between current and future security of supply. Two effects are at stake: on the one hand, inasmuch as foreign supply substitutes cannot expand quickly in a disruption, there is a motive for speeding up domestic production to reduce near-term economic losses; on the other, the scarcity value of domestic reserves is increased, providing an incentive for conservation to anticipate future emergencies. This trade-off has been analyzed by several authors (for example Stiglitz, 1977, Sweeney, 1977, Tolley and Wilman, 1977, Hillman and Van Long 1983, Hugues Hallet, 1984). A typical analysis postulates a range of hypothetical supply interruptions for a representative year. Using a conventional description of supply and demand, a comparison of the pre- and post interruption markets reveals the changes in prices, payments for imports, and consumer surplus that make up the economic costs of interruption. Weighted by the probability of interruption, this comparative statics provides an estimate of the expected costs of supply insecurity. As stock drawdown increases supply, the resulting reduction in the costs of interruption yields the estimate of the value of the reserve. However, as Lindsey (1989) shows by generalizing both the models of Tolley and Wilman (1977) and Hillman and Van Long (1983), the extraction rate of a domestic resource is extremely sensitive to the allocation mechanisms for supplies in a disruption.

Policy makers have rapidly recognized that the oil stockpiling strategy can be considered as a public good, and this leads naturally to a free-riding problem. In this perspective, a second group of models includes the analysis of strategic interaction among importing countries and exporters (Nichols and Zeckhauser, 1977, Crawford, Sobel and Takahashi, 1984, Devarajan and Weiner, 1987, Hogan, 1983). Policy coordination at the supra-national level could attenuate this kind of inefficiency. The European security of supply measures can be interpreted as a step in this direction.

The renewed interest by the California Energy Commission in the problem of the strategic fuels reserve (SFR) motivated the work by Ford (2005). The analysis is based on a computer model and simulates refinery disruptions of different sizes and durations. Inspired by Pindyck (2001), the author studies the impact of public storage designed to limit the increase in gasoline prices in the days following refinery disruptions. Ford's model provides an interesting estimate of the potential economic value of precautionary storage: the consumers' net benefit of an SFR in a single disruption (15-day outage of 150 thousands of barrels per day) amounts to over $400 million. However, the marginal convenience yield is modeled by a reduced functional form whose ability to accurately capture storage costs is unclear. This could cause significant errors on the optimal stockpiling strategy.

However useful these analyses may be, they ignore the question of how to reach any desired stock level and how to deal with uncertainty about the duration of the supply disruption. Efficiency loss of recommended policies may be underestimated. These aspects are of crucial importance in the European context, as well as in any region where domestic reserves are playing a minor role. In the most significant attempt to address these questions, Teisberg (1981) developed a dynamic programming model that allowed explicit consideration of the uncertainty of the duration of interruptions and the timing of stockpile management, to be accounted for by minimizing a US cost insecurity function due to oil import disruption. The model displays the time path of stockpile build-up and drawdowns under different assumptions, including the consideration of tariffs and quota policies in addition to stockpile management. The main result of Teisberg's analysis is that it is often not desirable to use all of the stockpile during an emergency; part of the stockpile should be saved in case the supply interruption continues into the next period. Similarly, with a very low stockpile, it may be best to build the stock even during a small interruption, as a hedge against the possibility of even greater losses during future periods. Teisberg (1981) assumes that in a finite time horizon the price path will hit a pre-defined back-stop value at which market insecurity is no longer a problem and remaining stocks will be sold. This hypothesis, appropriate in a long-term perspective, is not well suited to short-medium term crisis management that our work considers.

In a different vein, Wright and Williams (1982) emphasize the role of public storage in managing oil import disruption in a stochastic economy and its relationship with private stockpiling. However, the assumption of i.i.d. shocks used by Williams and Wright cannot capture the persistence supply crises are likely to exhibit.[3]

## 2 The model

Time is continuous. The economy starts in a state of abundance $A$ and passes irreversibly in state of crisis $C$. The switch from $A$ to $C$ follows a Bernoulli

[3] See Chaton, Creti and Villeneuve (2005) for a more detailed discussion of the Williams and Wright approach.

process with (publicly known) parameter $\lambda$.

Primary production and final consumption at a given date vary with the state $\sigma = A, C$ and the current price. For every state, the difference, per unit of time, between production and consumption (the excess supply function) is denoted $\Delta_\sigma[p]$ where $p$ is the spot price; this function measures the sensitivity of the economy to the crisis, comprising adaptation of demand (in particular fuel switching). Denoting the stocks by $S$, we have

$$\left.\frac{dS}{dt}\right|_\sigma = \Delta_\sigma[p]. \tag{1}$$

Excess supply function $\Delta_\sigma$ is increasing and has a unique finite positive zero in $R^*_+$, denoted by $p^*_\sigma$; this is the price at which the spot market would be balanced without recourse to stocks. Consistently with the terminology, we assume $p^*_A < p^*_C$.

Storage is competitive by hypothesis and exhibits constant returns to scale. Carrying costs consist of the opportunity cost of capital ($r$ being the riskfree interest rate) and a cost $c$ (per unit of commodity in stock and per unit of time).[4] Storers are assumed to be risk-neutral, so that the price equations will be driven by arbitrage considerations.[5]

Notation:

$t$ : time;
$\sigma$ : exogenous state, $A$ : Abundance; $C$ : Crisis;
$\lambda$: probability rate that state passes from $A$ to $C$;
$\Delta_\sigma[p]$ : excess supply for price $p$ in state $\sigma$ (per unit of time);
$p^*_\sigma$ : zero of $\Delta_\sigma$;
$S$ : stocks;
$c$ : marginal storage cost (per unit of time);
$r$ : riskfree interest rate.

**The equilibrium.** A *competitive equilibrium* starts at date 0, in state $A$, with some stocks $S_0$; it is a sequence of contingent prices $\{p_A[S,t], p_C[S,t]\}_{S\geq 0, t\geq 0}$ such that (1) all agents (consumers, producers, storers) are price-takers and use rational expectations; (2) in all states, conservation of matter imposes that excess supply equals the variation of the stocks.

[4] A more general structure is discussed in Subsection 7.1.

[5] To be more rigorous, we should rather write "quasi arbitrage", since speculators break even in expectation only.

Non-strategic behavior of the agents, strictly increasing excess supply functions, linearity of the storage technology, risk-neutrality, all these hypotheses suffice to ensure that the competitive equilibrium is Pareto optimal.[6]

For $S > 0$ and a time increment $dt$, the no-arbitrage equations read

$$p_C[S,t] + cdt = (1 - rdt)p_C[S + dS, t + dt], \tag{2}$$

$$p_A[S,t] + cdt = (1 - rdt)\left((1 - \lambda dt)\, p_A[S + dS, t + dt] + (\lambda dt)p_C[S + dS, t + dt]\right). \tag{3}$$

In the above equations, the LHS is the unit price plus stockholding cost in states $C$ and $A$ respectively. The RHS is the present expected unit value of the stocks after $dt$ has elapsed. Equation (3) incorporates the possibility of the price jump when the crisis occurs. After elimination of second order terms, we get

$$\frac{\partial p_C}{\partial S} \cdot \left.\frac{dS}{dt}\right|_C + \frac{\partial p_C}{\partial t} = rp_C + c, \tag{4}$$

$$\frac{\partial p_A}{\partial S} \cdot \left.\frac{dS}{dt}\right|_A + \frac{\partial p_A}{\partial t} = (r + \lambda)p_A - \lambda p_C + c. \tag{5}$$

State $C$ being irreversible, the crisis price is deterministic thus we will sometimes use equation (4) under the form

$$\frac{dp_C}{dt} = rp_C + c. \tag{6}$$

**Stationarity.** None of the model's parameters (interest rate, costs, crisis probability) depend on time. The stationarity of the economy allows us to eliminate time and to summarize the equilibrium by $\{p_A[S], p_C[S]\}_{S \geq 0}$. Keeping $S$ and $\sigma$ as the only relevant state variables, the price dynamics in (4) and (5) become

$$\Delta_C[p_C] \cdot \frac{dp_C}{dS} = rp_C + c, \tag{7}$$

$$\Delta_A[p_A] \cdot \frac{dp_A}{dS} = (r + \lambda)p_A - \lambda p_C + c. \tag{8}$$

# 3 Price and stock dynamics

We solve the model backwards. We show that once the crisis has occurred, the price stabilizes at $p_C^*$ after a finite period, whose duration increases with

[6] See for instance note 1 in Scheinkman and Schechtman (1983).

respect to the stocks at the moment of the crisis. We also show that, in state $A$, there is a target precautionary stock $S^*$, which is never attained. Comparative statics are exposed, and finally, we discuss the irreversibility hypothesis.

## 3.1 Drainage

**Price.** Once the crisis has broken out, it would be inefficient to conserve stocks longer than a certain duration. Indeed the present cost at date $t_0$ of storing gas units for a period of length $\delta$ is strictly positive (it approaches $p_C[t_0] + c/r$ as $\delta$ goes to infinity), but the present value of the benefit goes to zero (marginal benefits are bounded above by $p_C^*$, so the discount factor drives future benefits to zero as $\delta$ goes to infinity).

Assume that when the shock occurs, the stock is $S$. The crisis price starts at $p_C[S]$: the drainage duration $D[S]$ is the time required for the price to pass from $p_C[S]$ to $p_C^*$, when it follows equation (6). We can exclude $p_C[S] > p_C^*$, otherwise the price would increase forever as well as stocks (see equation 7), and this clearly cannot be an equilibrium. Let $\tau$ be the time elapsed since the beginning of the crisis and $p[\tau]$ be the price at that moment; we have

$$\begin{cases} \text{for } \tau \in [0, D[S]] : \ p[\tau] = (p_C[S] + \frac{c}{r}) \exp[r\tau] - \frac{c}{r}, \\ \text{for } \tau > D[S] : p[\tau] = p_C^*. \end{cases} \tag{9}$$

We find

$$D[S] = \frac{1}{r} \ln \left[ \frac{rp_C^* + c}{rp_C[S] + c} \right]. \tag{10}$$

**Stocks.** The storage dynamics between "dates" $\tau$ and $\tau + d\tau$ in state $C$ is determined by $dS = \Delta_C [p[\tau]] .d\tau$, where $p[\tau]$ follows (9). The total stocks variation since the beginning of the crisis is therefore given by

$$\Sigma = \int_0^{D[S]} \Delta_C [p[\tau]] \, d\tau. \tag{11}$$

Using equation (6) to change variables, we find

$$\Sigma = \int_{p_C[S]}^{p_C^*} \frac{\Delta_C [p]}{rp + c} dp < 0, \tag{12}$$

which depends only on exogenous parameters and on $p_C[S]$. Conservation of matter imposes that stocks are equal to what is going to be drained:

$$S = -\int_{p_C[S]}^{p_C^*} \frac{\Delta_C [p]}{rp + c} dp. \tag{13}$$

The LHS increases with respect to $S$ and the RHS decreases with respect to $p_C[S]$. We conclude that $p_C[S]$ is a decreasing function: larger stocks always need more time to be drained.

## 3.2 Stockpiling

During the state of abundance, the economy accumulates stocks ($\Leftrightarrow p_A[S] \geq p_A^*$). The maximum stock the economy can reach in the competitive equilibrium is called the target stock $S^*$ ($p_A[S^*] = p_A^*$). In reality, the crisis may occur before the economy has reached (or even got close to) the target. As $S$ approaches $S^*$, accumulation slows down; in other terms, $p_A[S]$ is a decreasing function. These properties are formally proved in Appendix A.1.

A salient property of the equilibrium is that the target stock is never reached in finite time. The price $p_A$ must converge continuously towards $p_A^*$ before the occurrence of the gas disruption. As $p_A$ covers half its difference with the target $p_A^*$, the variation rate of the stock per unit of time $\Delta_A$ is approximately halved (the derivative of excess demand at $p_A^*$ is not zero), meaning that the convergence speed $dS/dt$ is approximately halved. This implies that, whatever the proximity of the target, the duration to cover half the distance to the target is approximately constant, thus the target is never attained.

We define

$$\overline{p}_C = \left(\frac{r+\lambda}{\lambda}\right) p_A^* + \frac{c}{\lambda}. \tag{14}$$

At the target stock $S^*$, $p_A[S^*] = p_A^*$, therefore in case of crisis, the price jumps to $p_C[S^*] = \overline{p}_C$, which is the minimum crisis price consistent with accumulation in state $A$. The equation (13) gives the equilibrium target stock

$$S^* = -\int_{\overline{p}_C}^{p_C^*} \frac{\Delta_C\,[p]}{rp+c} dp. \tag{15}$$

The target stock is strictly positive if and only if $\overline{p}_C < p_C^*$; hence after rearrangement

$$S^* > 0 \Leftrightarrow \lambda > \frac{rp_A^* + c}{p_C^* - p_A^*}. \tag{16}$$

This means that the carrying costs ($rp_A^* + c$ per unit) must not exceed the expected earning ($\lambda(p_C^* - p_A^*)$ per unit). If, on the contrary, the difference between the two boundary prices is too small or if $\lambda$ is too small, then necessarily $S^* = 0$ and the crisis will simply cause a price jump from $p_A^*$ to $p_C^*$.

By plugging equation (14) into (10), we obtain the drainage time of $S^*$

$$D^* = \frac{1}{r} \ln \left[ \frac{\lambda}{r+\lambda} \frac{rp_C^* + c}{rp_A^* + c} \right]. \tag{17}$$

Obviously, $D^*$ is positive if and only if $S^*$ is positive. Equation (17), which depends only on the boundary prices $p_C^*$ and $p_A^*$, the interest rate and the unit cost, provides useful orders of magnitude. In practice, if $c$ is negligible with respect to the opportunity cost of the stock, then we can estimate that $S^*$ and $D^*$ are non null if

$$\frac{p_C^*}{p_A^*} > \frac{r+\lambda}{\lambda}. \tag{18}$$

With an interest rate of 10% and a “one-in-ten-years” crisis ($\lambda = 10\%$) some precautionary storage takes place if the ratio $p_C^*/p_A^*$ is larger than 2.

## 3.3 Comparative statics

As long as excess supply functions $\Delta_C$ and $\Delta_A$ remain unchanged, the impact of the model’s parameters $c$, $r$, $\lambda$ are unambiguous (see equations 15 and 17). A higher crisis probability requires a lower $p_C[S^*]$, so a longer time is needed for the price to reach $p_C^*$; this longer episode will drain a higher target stock $S^*$. For a higher $r$ or $c$, $p_C[S^*]$ shifts upward and the price increases faster, so $p_C^*$ is reached more rapidly. These effects cause shorter $D^*$ and smaller $S^*$.

Changes in the excess supply functions can be interpreted as the development of the transport infrastructure, by connecting the gas network to additional pipelines or by building LNG terminals. The overall impact on $S^*$ and $D^*$ is not clear-cut. For instance, a lower $p_C^*$ could be associated with either a more or a less elastic excess supply; in the former case, $S^*$ will decrease, whereas in the latter case no conclusion is warranted. To focus on level rather than elasticity effects, one can replace $\Delta_\sigma[p]$ by $\Delta_\sigma[p + \epsilon_\sigma]$ for some positive $\epsilon_\sigma$ (i.e. $p_\sigma^*$ replaced by $p_\sigma^* - \epsilon_\sigma$). Shifting $\epsilon_A$ increases $S^*$ and $D^*$, whereas $\epsilon_C$ plays in the opposite direction. Supposing the negative effect on $p_C^*$ to be much stronger than the one on $p_A^*$ (the marginal impact during the crisis could be much greater than the one we have under normal circumstances), one can intuitively estimate that the development of transport infrastructures reduces the need for precautionary storage. However, this is an empirical issue that this model cannot directly answer.

# 4 On the irreversibility hypothesis

In this section, we partially relax the irreversibility hypothesis, by extending the model in two directions: first, we consider a long but finite duration of the crisis, and second, we study the impact of "alerts" in the management of stocks.

By an appropriate specification of the excess supply function $\Delta_C$, as mentioned in Section 2, our model takes into account the short-term reactivity of the economy to the shock via demand curtailment or fuel switching; the introduction of a liberalized gas market should also favor gas transactions and offer interesting possibilities to overcome disruption problems. Supply crisis can also be solved by negotiating new contracts with gas producers and developing ad hoc transport infrastructure. This entails long and complex procedures, not to mention the time-lag between exploration of new gas fields and drilling. We model this idea by considering a long but finite duration of the crisis. The gas disruption that has hit Italy, France, Germany and Austria in January 2006, due to a shortfall in the Russian supply, provides a significant example in this direction. Due to gas withholding undertaken by Ukraine as a result of conflictual relationship with the main Russian gas producer, and exacerbated by a very cold winter in Eastern Europe, the significant reduction of the gas volume has posed serious problems in the stock management policies of importer countries.

The model also considers the impact of crisis forewarning. This approach can be useful when supply disruptions are caused by natural phenomena. For instance, the Katrina hurricane, that during the late Summer 2005 caused serious and persistent shutdowns of gas production units in the Gulf of Mexico, was forecasted to occur some days before its landfall. When the crisis is announced in advance, consumption can be precociously discouraged by price increases. This solution is particularly appealing in a context were the shortage occurs in the off-peak season but is forecasted to continue into the peak season, when gas demand for heating is very large.

## 4.1 Crisis of finite lenght

Assume that the agents know that the crisis will last a period of length $L$, after which the economy returns to a less critical state. When $L > D^*$, the accumulation and drainage dynamics behave *as if* the crisis were irreversible. If $L < D^*$, the maximum stock $S^L$ becomes smaller than $S^*$ and increases with the crisis duration $L$. If the economy has accumulated $S^L$, the prices range from $\overline{p}_C$ at the beginning of the crisis to $(\overline{p}_C + \frac{c}{r})\exp[rL] - \frac{c}{r} < p_C^*$ at the end of the crisis, when stockout is complete. Quite intuitively, storage is more effective at smoothing prices for short crises; however, if the shock occurs early, the accumulated stocks might be insufficient to last the whole duration of the crisis.

Whether the model's dynamics can be found using the irreversibility hypothesis can be checked ex post by comparing $L$ and $D^*$. For example, taking $r = .1, c = .01$, $p_A^* = 1$, and a "one-in-ten years" crisis ($\lambda = .1$), if $L \geq 5$ years, precautionary stocks are accumulated as if the crisis were irreversible whenever the ratio $p_C^*/p_A^*$ is not above 3.5. This suggests that our methodology is conclusive for realistic parameters.

## 4.2 Alert and crisis

We assume that the crisis is announced (the "alert") *before* it happens and stay as close as possible to the basic model. In the abundance state, the occurrence of an alert follows a Bernoulli process of parameter $\lambda$, and after a delay of $T$ time units, $T$ being perfectly known, the disruption takes place. We could think of $T$ as being a few weeks or months (up to now, we assumed $T = 0$).

There are two finite thresholds $\underline{T}$ and $\overline{T}$ with $\underline{T} < \overline{T}$ separating the three different regimes that we are going to describe (see Appendix A.2 for calculations).

Assume that the date of the crisis $t_0$ has always been known. As stockpiling too early is not profitable, there is a unique $\overline{T}$ such that at date $t_0 - \overline{T}$, the economy starts storing and does so until $t_0$; from then on, the stock is drained.

Clearly, if $T > \overline{T}$, accumulation starts after $T - \overline{T}$ time units spent in the alert state and continues until the crisis actually occurs. All the way along the realization of the random process, the price of gas evolves continuously. The price stays at $p_A^*$ until $\overline{T}$ time units before the crisis, and then starts to increase up to $p_C^*$. In the transition, stocks are piled up during the alert and drained once the crisis has hit the economy. Remark that accumulation accelerates as the date of disruption approaches, a difference with the basic model.

If $T$ is slightly below $\overline{T}$, as opposed to the previous regime, the price of gas jumps as soon as the crisis is announced and storers suddenly start accumulating. Quite intuitively, the jump increases as $T$ shortens. After the jump, storage follows a pattern similar to the one we described for $T > \overline{T}$. To avoid accumulation before the alert, this regime requires the price not to jump above $\overline{p}_C$.

The threshold $\underline{T}$ (with $0 < \underline{T} < \overline{T}$) is such that, for $T < \underline{T}$, the jump is high enough for some stockholding to take place *before* the alert. In that case, accumulation can be broken down into two phases. Before announcement of the disruption, the stock converges towards a target; during this phase, the price goes slowly down towards $p_A^*$. As soon as disruption is predicted, the price jumps and starts increasing as described in the previous paragraphs.

# 5 Policy issues

When the crisis breaks out, energy policy is likely to respond to political pressure. As Wright and Williams (1982) put it:

> [...] the oil industry has abundant reason to believe that there is some oil price at which government will intervene to control the realizations of oil drawn down from private storage in times of shortage, when profit-maximizing private storers and importers may well be branded as "speculators" or "price gougers". In fact, it may well be impossible for any administration credibly to guarantee against such action by itself or its successors.

Storers could either be submitted to price controls or even be forced to sell their stocks, at a low price, to some public agency. If this violation of property rights is anticipated, the accumulation process will be dramatically disturbed.[7] To mitigate this potential commitment failure, the Government may want, ex ante, to encourage accumulation by setting up a number of incentives. Other distortions (market power, import tariffs) can be analyzed in a similar way.

Our approach to these issues differs from Wright's and Williams' in two principal respects: in our view, crises are durable rather than i.i.d. shocks. Moreover, the constraints on prices we envisage are more general than the price caps they consider. Our assumptions enable us to show that the effect of expectations is so strong as to cause, under defensible scenarios, private storage market to collapse (zero stocks in equilibrium).

This political pressure can be summarized in general by some $\widehat{p}_C[S]$, the price of gas when the crisis happens with $S$ in stock. This first crisis price contains all the relevant information on subsequent management of drainage by the Government. The competitive accumulation dynamics with rational

[7] Consumers and industry can also maintain the option of switching to less convenient at a price that increases with rapidity of adjustment. Storage policies, and price interventions, affect these potential responses. Indeed the common policy of protecting consumers by preventing price increases means that the advantages of such flexibility are often lost in supply crises.

expectations, $\widehat{p}_A[S]$, is determined by

$$r\widehat{p}_A[S] + c = \lambda(\widehat{p}_C[S] - \widehat{p}_A[S]) + \Delta_A[\widehat{p}_A[S]] \cdot \frac{d\widehat{p}_A[S]}{dS}, \tag{19}$$

where the LHS is the total storage cost and RHS the expected benefit. We now envisage different formulations for $\widehat{p}_C[S]$. Obviously, competitive accumulation corresponds to $\widehat{p}_C[\cdot] = p_C[\cdot]$, where $p_C[\cdot]$ is defined in Section 3.1.

The Government may respond to political pressure by imposing a "fair" price $\widehat{p}_C$ independent of $S$. This policy has only one possible outcome: storage is completely discouraged. This effect is quite intuitive when $\widehat{p}_C \leq \overline{p}_C$ (see equation 14 for the definition of this threshold), since even with zero stocks, where the social marginal value of stored gas is maximum, the cost of purchasing speculative gas exceeds expected benefits. When $\widehat{p}_C > \overline{p}_C$, there is no limiting force to accumulation, thus any stock can be attained with positive probability as a consequence of a price bubble. This solution is eliminated by the fact that the economy cannot absorb unbounded stocks.

Total discouragement of storage is generalizable to a larger range of conjectures about $\widehat{p}_C[S]$, for example if the support of the function lies entirely below, or above, $\overline{p}_C$. Still, a category of functions can sustain positive stocks. Assume that $\widehat{p}_C[0] > \overline{p}_C$ and that $\widehat{p}_C[\cdot]$ is nonincreasing, piecewise continuous and left-continuous. Denote by $S_1$ the smallest stock such that

$$\begin{aligned} S &< S_1 \Rightarrow \widehat{p}_C[S] < \overline{p}_C, \\ S &> S_1 \Rightarrow \widehat{p}_C[S] \geq \overline{p}_C. \end{aligned} \tag{20}$$

Denote by $S_2$ the smallest point of discontinuity of $\widehat{p}_C[\cdot]$.

If $S_1 \leq S_2$, $S_1$ is the maximum stock attained, since it is clearly a stopping point of equation (19); remark that this implies $\left.\frac{d\widehat{p}_A}{dt}\right|_{S=S_1} = 0$. If $S_2 < S_1$, no stocks are accumulated. $S_2$ is not a stopping point because any additional purchase (at some price $\geq p_A^*$) will leave the price above $\overline{p}_C$, meaning that the marginal speculator earns money. However, this causes a sudden depreciation of the stock that hurts other storers: the anticipation of this unavoidable loss eliminates any equilibrium trajectory with accumulation.

Ex ante, the Government may see its own inconsistency as a constraint summarized by the constraint $\widehat{p}_C[S]$. To avoid complete market failure and preserve satisfactory stocks, some preventive measures need to be taken. One possibility is to rely entirely on a public agency. Subsidies too may be considered. Though the complete characterization of the constrained optimum is beyond the scope of this paper, we show that subsidies, even if they are discontinued as soon as the crisis occurs, are so powerful as to implement any accumulation path. Let $\widetilde{p}_A[S]$ be the desired accumulation policy under constraint $\widehat{p}_C[S]$; let $\mu_A[S]$ be the subsidy paid per unit of gas and per unit of time, conditionally on being in state $A$. If, for all $S$, the following relationship is verified

$$r\widetilde{p}_A[S] + c = \lambda(\widehat{p}_C[S] - \widetilde{p}_A[S]) + \Delta_A[\widetilde{p}_A[S]] \cdot \frac{d\widetilde{p}_A[S]}{dS} + \mu_A[S], \qquad (21)$$

then the subsidies $\mu_A[S]$ implement $\widetilde{p}_A[S]$ as a competitive equilibrium. In practice, the cost of these subsidies has to be balanced with the efficiency gains.

# 6 Policy evaluation

The preceding section suggested the importance of welfare evaluation in second best economies. To do so, clarifications on the behavior of the economic agents are required. The simplest one is to consider a representative agent whose intertemporal utility function valorizes gas consumption and a separable numéraire that could be labor. Leaving aside uncertainty for the moment, the objective can be written as

$$\int_0^{+\infty} (u_\sigma[q_t] - m_t)\, e^{-rt} dt \qquad (22)$$

where $u_\sigma$ is a state dependent, increasing and concave utility, $q_t$ is date $t$ gas consumption and $m_t$ is date $t$ expenditure. We also assume that producers' technology can by aggregated at $t$ by a state dependent convex cost function $C_\sigma[q_t]$. In the medium term, the infrastructure is given and the exhaustion horizon is too far in the future to substantially affect supply.

For a given price $p$, final demand is $u_\sigma'^{-1}[p]$ and primary production is $C_\sigma'^{-1}[p]$, thus excess supply functions can be expressed

$$\Delta_\sigma[p] = C_\sigma'^{-1}[p] - u_\sigma'^{-1}[p]. \qquad (23)$$

The instantaneous surplus can be computed as

$$W_\sigma[p_t] = W_\sigma^* + \int_{p_\sigma^*}^{p_t} \Delta_\sigma[p]dp - p_t\Delta_\sigma[p_t] - cS_t, \qquad (24)$$

where $W_\sigma^*$ denotes the surplus at $p_\sigma^*$. Given that gains or losses made by storers are pure redistribution, the only impact of storage on the surplus is the instantaneous cost $cS_t$ where $S_t$ is the stock at date $t$.

A given policy, whether optimal or not, can be represented by the price functions $\{\widetilde{p}_A[S], \widetilde{p}_C[S]\}_{S\geq 0}$ . The expected intertemporal surplus it generates, given an initial state $\sigma_0$, a stock $S_0$ at date 0 and the stockholding dynamics, is

$$\begin{aligned} V_{\sigma_0}[S_0] &= E\int_0^{+\infty} W_{\sigma_t}[\widetilde{p}_{\sigma_t}[S_t]]e^{-rt}dt, & (25)\\ \text{s.t. } \frac{dS_t}{dt} &= \Delta_{\sigma_t}[\widetilde{p}_{\sigma_t}[S_t]], & (26)\end{aligned}$$

where $\sigma_t$ is the (random) state at date $t$.

Remark that as the Bernoulli process driving the evolution of $\sigma_t$ is exogenous and time independent, the terms in the value function (25) comprising $W_A^*$ and $W_C^*$ are identical whatever the policy evaluated. This is extremely convenient since these numbers are not calculable without specific assumptions on $u_\sigma$ and $C_\sigma$, which we want to avoid. The value of a given policy can be written as

$$V_{\sigma_0}[S_0] = \quad (27)$$

$$V_{\sigma_0}^0 + E\int_0^{+\infty}\left(\int_{p^*_{\sigma_t}}^{\widetilde{p}_{\sigma_t}[S_t]} \Delta_{\sigma_t}[p]dp - \widetilde{p}_{\sigma_t}[S_t]\Delta_{\sigma_t}[\widetilde{p}_{\sigma_t}[S_t]] - cS_t\right)e^{-rt}dt,$$

$$\text{s.t. } \frac{dS_t}{dt} = \Delta_{\sigma_t}[\widetilde{p}_{\sigma_t}[S_t]], \quad (28)$$

where $V_{\sigma_0}^0$ denotes the (unknown) value of the no storage policy. The calculation of $V_{\sigma_0}[S_0] - V_{\sigma_0}^0$ only requires knowledge of the corresponding price functions, the $\Delta_\sigma$ and the stochastic process. This general method is used in the following example.

## The linear case

To illustrate the model, we assume linear differences between demand and supply:

$$\Delta_C[p_C] = bp_C - a \; ; \; \Delta_A[p_A] = \beta p_A - \alpha. \quad (29)$$

The reference prices are $p_C^* = a/b > p_A^* = \alpha/\beta$. Figure 1 illustrates the supply disruption in the linear case; the stock variation is positive when resources are abundant and negative during the crisis.

We describe and compare alternative policies:

1. No storage;

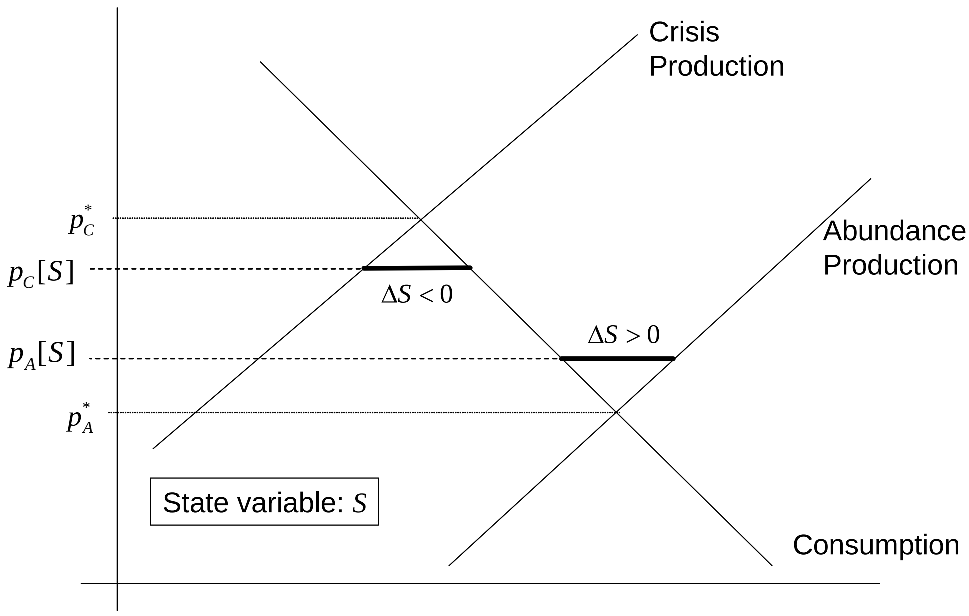


Figure 1: Supply disruption in the linear case.

2. The competitive/surplus maximizing storage;
3. "Simple" policies: fixed accumulation and drainage rates are defined by regulation as well as a maximum stock $S_{\max}$; these rates determine fixed prices during accumulation on the one hand and drainage on the other.

We take the following parameters:

| Table 1: Parameters | | | | |
|---|---|---|---|---|
| Costs | $r = .1$ | $c = .01$ | | |
| Excess demand | $b = 1$ | $a = 5$ | $\beta = 5$ | $\alpha = 5$ |
| Bernoulli parameter | $\lambda = .1$ | | | |

Time unit is the year, quantity unit is arbitrary.

**The optimum.** Using equation (15), we find the optimal target stock

$$S^* = \frac{bc + ar}{r^2} \ln\left[\frac{\lambda}{r+\lambda} \frac{rp_C^* + c}{rp_A^* + c}\right] + \frac{b}{r}\left(\left(\frac{r+\lambda}{\lambda} p_A^* + c\right) - p_C^*\right). \tag{30}$$

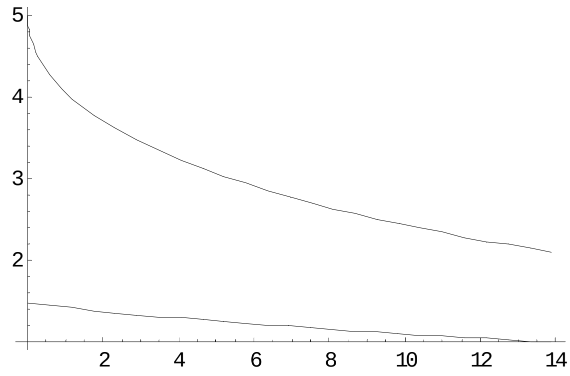


Figure 2: Equilibrium prices as functions of $S$ in states A (below) and C (above).

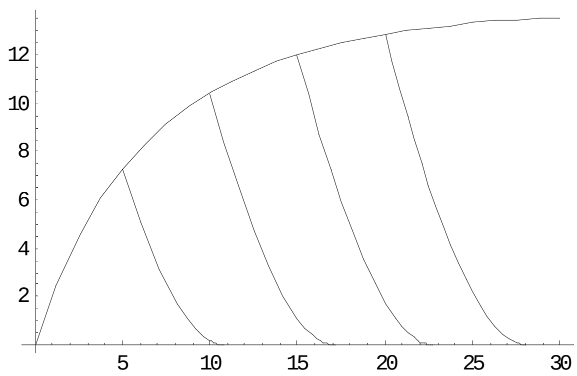


Figure 3: Stocks as a function of time with various crisis dates.

Drainage time $D^*$ is determined by the explicit equation (17); $p_C[S]$ and $V_C^*[S]$ can be explicitly calculated (expressions involve Lambert's $W$ function); $p_A[S]$ and $V_A^*[S]$ are solved numerically.

Let the economy start at $t = 0$ and $S = 0$ in state $A$. As long as the state is $A$, stocks are gradually piled up to approach $S^* = 13.88$ and the price decreases toward $p_A^* = 1$. In Figure 2, we show prices as a function of the stocks. Figure 3 depicts accumulation and drainage for alternative scenarios, where the shock occurs (unexpectedly) at dates $t = 5$, 10, 15 or 20. Drainage of the stocks takes $D^* = 8.4$ years at most.

**"Simple" policies.** $V_A[0]$, the value of a simple policy at zero stock, can be expressed as an explicit function of the accumulation and drainage rates, and $S_{\max}$. We have calculated numerically the surplus maximizing member of

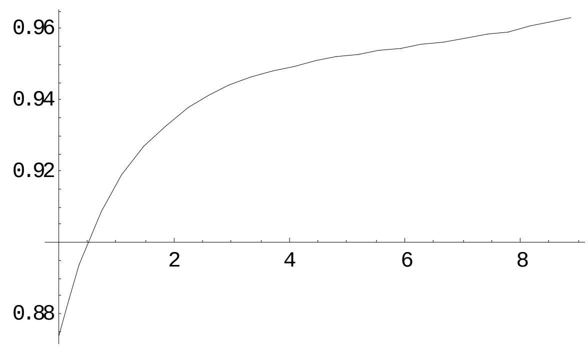


Figure 4: Relative value of constrained optimal policy in abundance state.

this family. We find $S_{\max} = 8.89$, $p_A = 1.30$ for $S \in [0, 8.89)$ and $p_A = p_A^* = 1$ once $S_{\max}$ is attained (accumulation stops suddenly, conversely to the optimal case), $p_C = 3.34$ (approximately 33% lower than $p_C^*$) for $S \in [0, 8.89)$ and $p_C = p_C^* = 5$ when stocks are empty.

Figure 4 displays the relative value of this constrained optimal policy, as a function of the stocks, in the abundance state, i.e.

$$\frac{V_A[S] - V_A^0}{V_A^*[S] - V_A^0}. \tag{31}$$

Over the common range ($S \in [0, 8.89)$), the difference is attenuated as stocks increase. At zero stock, the constrained policy captures 87% of the potential surplus; gains increase very fast at the beginning of the accumulation (at $S = 2$, i.e. 22% of $S_{\max}$, 70% of the initial difference is recouped). At $S_{\max}$, 96% of the maximum surplus are captured. To understand this effect, one should consider that as $S$ increases, the inefficiency of the constrained *accumulation* strategy disappears from policy evaluation, whereas the weight of inefficient crisis management is about the same (the transition probability is constant). For this reason, the comparison at zero best summarizes the difference between those two scenarios.

To show the sensitivity of welfare to suboptimal stock management, we evaluate another simple policy, where we arbitrarily set $S_{\max}$ at $S^*$ and we impose a twice larger accumulation rate and a twice slower drainage rate than in the previous constrained optimum. As Figure 5 shows, at zero stocks and up to $S = 0.8$ approximately, the policy does even worse than no storage at all (ratio starts at $-37\%$). Not only the price at which gas is paid to implement the policy is very high, but the economy has to sustain excessive reserves. At the target stock, the sub-optimal policy retrieves nearly 70% of

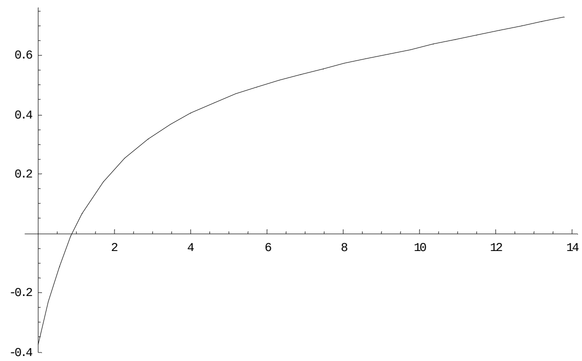


Figure 5: Relative value of suboptimal policy in abundance state.

the gains with respect to the optimum. As in the previous case, but to a lesser extent, having reached the target stock masks the inefficiency of the accumulation rule.

# 7 Extensions

## 7.1 Injection and release costs

The analysis can be extended to the case where the costs of injecting and releasing gas are non negligible. Denote unit injection cost by $i$ and unit release cost by $s$ (expressed in $\$.\mathrm{m}^{-3}$). Assume that in each state $\sigma = A, C$, and for any stock level $S$, there are markets for the gas *outside* and *inside* the reservoir, the prices being respectively $p_\sigma[S]$ and $p_\sigma^I[S]$. The market equilibrium between outside and inside gases implies that, whenever $S > 0$,

$$p_A[S] + i = p_A^I[S] \text{ and } p_C[S] = p_C^I[S] + s. \tag{32}$$

The structure of the system of equations is preserved, with $p_\sigma^I$ replacing $p_\sigma$. Arbitrage conditions (7) and (8) become

$$\Delta_C[p_C^I + s] \cdot \frac{dp_C^I}{dS} = rp_C^I + c, \tag{33}$$

$$\Delta_A[p_A^I - i] \cdot \frac{dp_A^I}{dS} = (r + \lambda)p_A^I - \lambda p_C^I + c. \tag{34}$$

Remark that the excess supply functions are shifted, thus boundary conditions are

$$p_C^I[0] = p_C^* - s, \tag{35}$$

$$p_A^I[S^*] = p_A^* + i. \tag{36}$$

The range of $p^I_\sigma$ is narrower than that of $p_\sigma$ : the minimum is higher, the maximum is lower. As a result, the condition ensuring positivity of the target stock is more restrictive, i.e.

$$p^*_C - s > \left(\frac{r+\lambda}{\lambda}\right)(p^*_A + i) + \frac{c}{\lambda}. \tag{37}$$

The calculation of $S^*$ and $D^*$ makes use of the shifted excess supply functions and boundary prices.

## 7.2 Limited storage capacity

Gas is mostly stored in depleted fields and aquifers; the development of such facilities is naturally limited. If the capacity devoted to precautionary storage $K$ exceeds $S^*$ previously calculated, then the unconstrained solution remains valid; otherwise, the maximum stock is constrained to equal $K$, which in turn affects price trajectories and the value of storage facilities.

During the crisis, $p_C[S]$ is unchanged compared to the unconstrained case. Reserves are gradually drained, meaning that the storage price, under competitive assumption, remains fixed at the marginal cost $c$. In the abundance state, the price function $p^K_A[S]$ depends on $K$ : the accumulation process must stop when capacity is saturated, therefore $p^K_A[K] = p^*_A$. The storage price is also $c$ as long as some capacity remains vacant and when $K$ is attained, it jumps to $\pi^K_A > c$, with

$$\pi^K_A = \lambda(p_C[K] - p^*_A) - rp^*_A. \tag{38}$$

The rent $\pi^K_A - c$, captured by the owners of the storage capacity, balances the carrying costs of an unchanging stock with its expected benefits.

Storage units gain value as $K$ diminishes. This combines two effects: the smaller $K$ becomes, the larger $\pi_A$, and also the shorter the time before saturation will be. The first effects, the monotonicity of $\pi_A$, derives directly from the monotonicity of $p_C[K]$. The second effect is shown in Appendix A.3.

# 8 Conclusion

We developed a model of optimal stockpiling and reserve duration to face up to a potential irreversible supply shock. To conclude, we underline some results that might prove useful in the context of the gas industry:

- Whether precautionary stocks should be accumulated is calculated, knowing the potential minimum and maximum prices, the carrying costs and the probability of crisis.

- The optimal target stock and the corresponding drainage time increase with the probability of a shock and decrease with the unit cost of storage and the interest rate.
- Additional gas pipelines are likely to decrease the optimal depletion period and thus the need for precautionary stockpiling.
- The model appropriately describes stock dynamics and equilibrium also when the crisis, more realistically, has finite length.
- Announcement of the crisis matters. Differentiating between the alert and the crisis as such, we illustrate the effect of the delay between these two events on the structure of the equilibrium path.
- The cost structure and the availability of limited storage capacity do not alter the main properties of the model.

Precautionary storage regulation should be flexible enough to accommodate changes in expectations and in the economic environment, and should supplement other means, such as long-term contracts, interruptible demand, spot and forward markets, to safeguard security of natural gas supply as recommended by the recent European directives. Our policy analysis is based on a complete understanding of the optimum as well as of constraints that may hinder its implementation. Indeed, the optimal rules (accumulation and drainage) we characterized may present practical or political difficulties, like expropriation threats that discourage efficient storage. Imperfect security obligations may be better or worse than no storage. Alternative scenarios (obligation to hold gas stocks equivalent to $x\%$ of the annual supply, to meet 1 in $x$ years peak day demand and 1 in $y$ years winter duration, etc.) and different assumptions of regulation of final and transportation prices can be rationalized—or eliminated—by calibrating the parameters of the model.

# A Appendix

## A.1 Target stock and accumulation dynamics

$p_C[S]$ being known, $p_A[S]$ is determined by the ODE (8). Denote by $S^*$ the smallest $S \geq 0$ such that $dp_A/dt = 0$, if there is one; if there is none, consider that $S^*$ is infinite. We have two cases: over the interval $[0, S^*)$, either (i) $dp_A/dt > 0$ or (ii) $dp_A/dt < 0$.

Case (i). Equation (8) can be rewritten

$$\frac{dp_A}{dt} = (r+\lambda)p_A[S] - \lambda p_C[S] + c > 0. \tag{39}$$

Qualitative analysis of this equation is straightforward: $p_C$ being decreasing, the variation rate of $p_A$ accelerates as time passes. This implies that we would have a price bubble equilibrium in which any level of stock is attainable with positive probability (any duration of state $A$ has a positive probability), which is physically impossible.

Case (ii). $S^*$ is a stopping point of the differential equation defining $p_A$. This characterizes the properties of $p_A$ : if $S^*$ is strictly positive, then $p_A$ decreases over $[0, S^*]$ and $p_A[S^*] = p_A^*$, thus accumulation slows down as the target is approached.

## A.2 Alert duration thresholds

**Derivation of $\overline{T}$.** Assume that $T$ is very large. Once the economy is in alert, uncertainty vanishes and the price follows the differential equation

$$\frac{dp}{dt} = rp + c \tag{40}$$

as long as $S > 0$, i.e. during the accumulation phase and drainage (no discontinuity at the instant the crisis occurs). Let $\overline{p} \in (p_A^*, p_C^*)$ be the price reached when the crisis occurs. Using the same change of variable as in the text, we know that conservation of matter implies

$$\int_{p_A^*}^{\overline{p}} \frac{\Delta_A[p]}{rp+c} dp + \int_{\overline{p}}^{p_C^*} \frac{\Delta_C[p]}{rp+c} dp = 0. \tag{41}$$

$\overline{p}$ is clearly unique. Remark that $\overline{T}$ is the time required for the price to pass from $p_A^*$ to $\overline{p}$, i.e.

$$\overline{T} = \frac{1}{r} \ln \left[ \frac{r\overline{p} + c}{rp_A^* + c} \right]. \tag{42}$$

**Derivation of $\underline{T}$.** For all $T$ between $\underline{T}$ and $\overline{T}$, the first post-alert price $p_A^T$ must be such that, prior to alert, no storage takes place, i.e.

$$p_A^T < \left( \frac{r+\lambda}{\lambda} \right) p_A^* + \frac{c}{\lambda}. \tag{43}$$

Let $\widetilde{p}$ be the price of gas at the instant the crisis occurs; it is uniquely defined by the conservation of matter equation

$$\int_{p_A^T}^{\widetilde{p}} \frac{\Delta_A[p]}{rp+c}dp + \int_{\widetilde{p}}^{p_C^*} \frac{\Delta_C[p]}{rp+c}dp = 0. \tag{44}$$

This provides us with a strictly decreasing relationship between $\widetilde{p}$ and $p_A^T$. The time required for the price to pass from $p_A^T$ to $\widetilde{p}$ is

$$\widetilde{T} = \frac{1}{r}\ln\left[\frac{r\widetilde{p}+c}{rp_A^T+c}\right]. \tag{45}$$

We conclude that $\widetilde{T}$ decreases as $p_A^T$ increases in the interval $[p_A^*, \left(\frac{r+\lambda}{\lambda}\right)p_A^* + \frac{c}{\lambda}]$. In particular, $\underline{T}$ corresponds to $p_A^T = \left(\frac{r+\lambda}{\lambda}\right)p_A^* + \frac{c}{\lambda}$.

For any $T$ smaller than $\underline{T}$, some accumulation takes place before the crisis is announced. See main text for a description of the equilibrium path in that case.

## A.3 Monotonicity of the scarcity rent

$p_A^K$ follows ODE (8), with boundary condition $p_A^K[K] = p_A^*$. As function $p_C$ is independent of $K$, the Cauchy-Lipschitz theorem implies that the price functions for two different capacities below $S^*$ never cross. Thus for all $S \in [0, K]$ and $K < K'$, $p_A^K[S] < p_A^{K'}[S]$ with both functions decreasing. We now show that the time $T_K$ needed for the price to pass from $p_A^K[0]$ to $p_A^*$ is longer the larger is the capacity $K$. Using equation (8), we have

$$T_K = -\int_{p_A^*}^{p_A^K[0]} \frac{dp_A}{(r+\lambda)p_A - \lambda p_C[p_A^{K(-1)}[p_A]] + c}. \tag{46}$$

Given the monotonicity of $p_A^K$ with respect to $K$, the above sum with a larger $K$ integrates a function of higher absolute value over a longer interval. This gives us the announced result.

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