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Opening the Retail Electricity Markets: Puzzles, Drawbacks and Policy Options*

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Abstract

The Italian electricity retail market is fully liberalized since 2007, allowing all households to choose between a regulated tariff and those offered in the free market. However, as of 2015, almost 70% of households remains with the regulated contract and only 3.6% moves every year to the free market. In this paper we first analyze retailers' strategies identifying the best and worst offers and the average bill on the free market. We find significant potential gains but also losses when switching from the regulated tariff to the free market. Then we build up a sequential search model that extends Janssen et al. (2005) to explain this evidence. Consumers have zero (shoppers) and positive (non-shoppers) search costs. These latter receive upward (pessimistic) or downward (optimistic) biased signals of their current regulated price. We obtain a rich set of mixed strategy equilibria with continuous support and, in some cases, an atom. The equilibria are characterized by price dispersion, different level of participation of non-shoppers of either type and some contracts more costly than the regulated one. Search costs and perception bias are key parameters in comparative statics, with policy implications to improve market performance. Finally, by mid 2019 the Government has planned to lift the regulated tariff. We use the model to predict that if the market before liberalization is large enough, prices are expected to raise once dropped.the regulated contract.

Keywords: Search costs, liberalized retail markets, price dispersion, gains and losses from switching

JEL: L13, L15, L94

1 Introduction

The liberalization process in the European electricity markets has entered into its third decade after the first Directive 96/92/CE. From the very beginning,

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one of the building blocks has been the opening of the demand side, by entitling customers the right to choose their own provider. Eligibility has been initially granted to large industrial energy-intensive users and then extended, with a different timetable in the Member countries, to lower consumption classes up to including all households. The retail electricity markets, however, have shown, in particular for households, low switching rates and a relatively slow pattern towards active consumers' participation. Several studies have explored the causes of low consumers' engagement, suggesting that a combination of search and switching costs, low expected gains and cognitive bias may account for this evidence.

Italy has followed the liberalization process reforming the electricity markets in 1999 and opening the retail markets for all users by July 1rst 2007. Households can maintain their previous provision contract with the local distributor under a regulated price that is quarterly adjusted by the Authority, or they can search in the free market for alternative offers. The regulator publishes every two years a report that provides evidence on the evolution of retail energy markets. According to the last release in March 2017, covering the period 2014-15, the regulated segment still accounts for about 68% of households and 64% of their consumption, although this share has been declining in the last years. As of 2015 the exit rate from the regulated tariff was just below 4% and declining with respect to the previous years. Interestingly, a non negligible percentage of customers switch back from the free market to the regulated one, suggesting that some offers in the free market may be less convenient than the default contract. This fact does not seem to be a marginal episode. The Italian regulator in its latest monitoring report confirms a puzzling evidence already found in the 2012-3 period. Households subscribing contracts in the free market pay on average a price higher than the regulated one. This evidence suggests that moving to the free market may expose small customers to a frustrating experience.

This paper aims at exploring from an empirical and theoretical perspective the main features of the Italian retail electricity market after the liberalization. We first provide evidence, adapting the methodology of the Energy Market Investigation (2015) of the UK Competition and Market Authority, on retailers' strategies in the free market and quantify the potential gains and losses from switching, corresponding to the difference in the electricity bill when moving from the regulated contract to the offers available in the free market. We find that about 60% of the contracts proposed to new clients are more convenient than the regulated one, but the other 40% are more expensive and the average bill computed on the available contracts is slightly higher than the default contract. We then look at the best and worst offers considering different customer's profiles in terms of annual consumption, peak/off peak allocation of consumption and preferences for certain contractual features. We find that significant gains, up to 21%, can be realized, in particular for low consumption classes, but customers may also suffer large losses, of the order of 30% of the annual bill, if picking up certain contracts. Our evidence, based on the contracts offered in the free market, illustrates the commercial strategies of firms, and shows that rip-offs, and not only bargains, are proposed to the households that may consider to switch from the regulated contract to the free market.

We move then to develop a model that may account for these stylized facts. While price dispersion and low participation, despite potential gains from switching, may be consistent with search costs, a standard model of sequential search cannot account for retailers' strategies that offer contracts more costly than the regulated price. We build on Janssen et al. (2005) model of sequential search with zero (shoppers) and positive (non-shoppers) search costs by introducing a second dimension of heterogeneity among non-shoppers. These latter are assumed to receive a biased signal of the true regulatory price they are currently paying, with an upward (high type) or downward (low type) distortion. This perception bias may easily emerge since the price, that is quarterly updated by the regulator, is hard to recover from the structure of the tariff and from the electricity bill. Moreover, this assumption is consistent with some evidence from consumer surveys on the degree of information of small customers in Italy.

The two dimensions of heterogeneity on the customers' side enrich significantly the set of mixed strategy equilibria compared with the original model, with shoppers and at least some high type non-shoppers participating in the free market. In some of these equilibria the most expensive price is above the regulated one and is subscribed by the high type non-shoppers that randomly quote it. These equilibria are characterized by a continuous support up to the regulated price and an atom at the reservation price of high types, a feature that does not take place in the Janssen et al. (2005) model. Moreover, some of the equilibria display unconventional comparative statics with respect to search costs and perception bias.

We argue that our set up is able to reproduce the puzzling evidence found in the first part of the paper and in reports on the Italian retail market. The model also suggests some policy options. First, in the debate on retail energy markets the focus is usually on improving transparency and reducing search costs to sample *new* contracts. Our model shows that a clean information on the *current* bill is as important in determining the market equilibria. Secondly, by mid 2019 the Italian government has planned to lift the regulated tariff, moving all households to the free market, a policy measure that is widely discussed also in other Member Countries of the European Union. The price adjustment that will take place depends on the equilibrium configuration at the time of the lifting and on the properties of the market equilibria at the time the default contract will be removed. Hence, having a clear understanding of the present features of the retail market is crucial to identify the more effective policy measures that may ease the transition. We use the insights of our model and Janssen's to predict that, if the market before full liberalization is large enough, prices are expected to increase.

1.1 Relation to the literature

Our paper contributes, and is connected, to two different strands of literature. First, we offer new evidence on the development of retail electricity markets and the behavior of consumers when liberalization starts. Secondly, our theoretical model has some new distinct features and results within the literature on consumers' search and market equilibria.

Regarding the first topic, there is an important body of empirical work studying *consumer engagement* in energy retail markets in Europe and the US (Crampes and Waddams Price, 2017). A first set of papers studies the determinants of consumers' search and switching activities starting from survey data. Expected net gains from search and switching and their persistence are generally found to be an important factor in consumers' decision to be active in the market (Flores and Waddams Price, 2013, Giulietti et al., 2005, Ek and Soderholm, 2008). On the other hand, perceived or actual cost of search, processing information and switching costs are found to limit consumer activity (Ek and Soderholm, 2008). Loyalty is another factor that may slow down consumer engagement creating a status-quo bias despite potential gains from switching (Gamble at al., 2008, Ek and Soderholm, 2008, Hortacsu et al., 2015). Ek and Soderholm, 2008 and Bladh, 2005 find that socio-demographic variables as income and education have a significant effect on market participation. Hence, empirical studies have highlighted a persistent consumers' inertia in retail energy markets despite large potential gains available in the market.

Several papers have estimated the perceived *gains from switching*, usually obtained through consumer surveys (Flores and Waddams Price, 2013, Waddams Price et al., 2013). Actual gains, instead, are computed as the effective difference between the current tariff and the best available option (Hortacsu et al., 2015). One major study of the effective gains has been provided by the UK Competition and Markets Authority (CMA) as a part of its 2015 Energy Markets Investigation. The CMA has collected a unique dataset on the tariffs available in UK retail energy markets, with an unprecedented level of detail. [1] The Authority calculated annual bills for different offers and consumption levels. Then, it defined a number of switching scenarios, where consumer choice set is restricted to tariffs with certain characteristics (supplier, payment method, price structure and contract length) and calculated and selected tariffs relative to it. Gains are calculated as the difference between the starting tariff and the lowest tariff belonging to the relevant scenario; they are then aggregated and their distribution is computed in each scenario. The CMA finds that internal switching within tariffs offered by the Big 6 guarantees gains from 4% to 6% of the annual bill (findings are similar for external switching). When the tariff choice is unrestricted, average gains reach 14% of the current bill. Our paper adapts the methodology of the CMA inquiry to the Italian electricity retail market and investigates the potential, rather than the effective, gains in the free market with respect to the default regulated tariff that household can choose, as well as the potential losses if "wrong" tariffs in the free market are selected.

[1] More specifically, for the period under analysis (2012-2015) the CMA has (1) firm level data on each tariff offered by the six greatest retail electricity suppliers, with details on the number of customers subscribed and the specifics of the contract; (2) estimated distributions of annual consumption for different families of tariffs offered by the Big 6; (3) data on all the tariffs offered in the market, obtained from a British PCW, Energylinx.

The next Section reports our main results.

The second stream of literature relevant for our research is on search costs and market equilibria. [2] Price dispersion and the effects of search costs on market equilibria are at the core of this field of research. Several dimensions have been explored, including the case of homogeneous (Varian, 1980, Burdett and Judd, 1983) and differentiated products (Wolinsky, 1984 and 1986, Anderson and Renault, 1999), buyers' (Stahl, 1989 and 1996, Janssen et al., 2005, Moraga-Gonzalez et al., 2016) and sellers' heterogeneity (Reinganum, 1979, Bar Isaac et al., 2012) and different search technologies as simultaneous search (Burdett and Judd, 1983), sequential search (Stahl, 1989, Janssen et al., 2005) and, more recently, ordered search (Armstrong, 2016, Anderson and Renault, 2016, Arbatskaya, 2007). Consumers' inertia and the failure to select the best price has been explored also in the recent literature that follows a behavioral approach of consumers' choices (Grubb, 2015).

Our paper uses a framework similar to Janssen et al. (2005) but enriches the heterogeneity of buyers, that are not only characterized by different search costs but also by different reservation prices. This way we obtain a much richer set of equilibria than in the original paper, including those in which the mixed strategy has a continuous support and an atom at the upper bound.

The paper is organized as follows. In Section 2 we present our evidence on gains and losses from switching as arising from an analysis of the available offers in the free market in the first quarter of 2017. In Section 3 we build up a model of costly sequential search and market equilibria. Section 4 and 5 report the equilibrium analysis, Section 6 develops comparative statics, Section 7 applies the model to the effects of lifting the regulated price. Conclusion follow in Section 8. Appendix I illustrates the methodology of the empirical analysis, Appendix II provides all the proofs and Appendix III discusses some properties of the reservation price of consumers.

2 The Italian retail electricity market: empirical evidence and puzzles

In this section, we carry out an empirical analysis of the potential gains and losses that would be available to those Italian consumers willing to switch from the regulated retail tariff to a free market offer. The regulated tariff is a default two-price contract where the unit price is set by the Italian regulator every 3 months. Maximum gains and losses in the free market then can be computed comparing the electricity bill of the regulated contract with the best and worst alternatives among those available in the free market.

[2] See Anderson and Renault 2017 and Baye et al. 2006 for excellent surveys.

2.1 Data

Since the regulator does not release data on the consumption level and time profile of individual subscribers of the regulated contract, we have to create different household profiles and, for each of them, compute the annual bill according to the regulated contract and to those offered in the free market, selecting the best and worst contracts.

The electricity bill is composed by fiscal and regulatory components, which are common to all contracts, and by the unit energy price and a fixed commercialization charge, which are freely set by market operators. For a given contract the annual bill depends on the fixed and common components, on the power installed, on the total annual consumption and, in case of a two-price tariff, on the allocation of consumption between peak and off-peak hours. We consider contracts for residential domestic consumers with a *power installed* not larger than 3 KW, that account for 77% of households subscribing the regulated contracts. We borrow the classification of *annual consumption* in 6 classes adopted by the Italian regulator in its surveys and use for each class the level of average annual consumption of the corresponding customers with an installed power of 3 KW as reported by the Regulator.[3] Finally, we define 3 types of consumers according to the *peak/off peak allocation* of consumption (85%, 70% or 50% of electricity usage in the evening and night time). Hence, total consumption and peak and off peak allocation determines 18 different groups of consumers. These are further distinguished according to their preferences for contractual features.

Comparing contracts in the free market, indeed, they differ under several dimensions, as a fixed/variable price mechanism, single or double price and methods of payments. Regarding the *contractual features* we define 4 different scenarios: consumers interested in all available contracts, only in single-price offers, in contracts with a 12 month fixed energy price and in contracts that allow payment through postal paying-in slips.

We retrieved data on retail bills through *TrovaOfferte*, the PCW created and controlled by the regulator AEEGSI [4]. Firms active on the market voluntarily communicate data on prices, discounts and various tariff characteristics and the PCW uses both internal control and a system of feedback to monitor quality and precision of the information. To calculate potential gains and losses, we use data on the first quarter of 2017.

[3] The Regulator uses also a seventh class of annual consumption that however is too high for customers with an installed power not larger than 3 Kw. We retrieve data on consumption from the regulator's annual report on the state of the electricity market. We use data from the year 2015, the most recent with consumption data available.

[4] According to the data reported in the AEEGSI's report Monitoraggio Retail from 2015, the suppliers that share tariff prices with *TrovaOfferte* cover 90% of the total volumes exchanged on the electricity market.

2.2 Results

We report here results for resident consumers with 3 Kw of power installed, the most frequent among domestic consumers [5]. Our analysis highlights three interesting facts. First, we find that, at any level of annual consumption, the average bill[6] computed on all the available offers in the PCW is slightly higher than the corresponding regulated one. Table 1 reports the average bill for the six classes of consumption and the corresponding bill of the regulated contract. We can observe that picking up a contract randomly in the free market implies a more expensive bill than sticking on the default contract.

Table 1 - Average bill on the free market and with the regulated contract

consumption classes	C1	C2	C3	C4	C5	C6
free market (average)	191,8	300,0	407,8	560,1	788,8	1125,2
regulated contract	189,2	294,5	400,4	550,8	777,2	1109,0
%Δ	1,38	1,86	1,86	1,69	1,50	1,45

Secondly, consumers could obtain significant gains by switching from the regulated tariff to the best available options in the free market. This fact contradicts a common wisdom that low participation would derive from too limited gains from switching.[7] Unexploited and significant gains from switching may point to the presence of significant consumer inertia, that is confirmed by the moderately low switching rates observed in the retail market. Hence, our analysis suggests consumers may face search or switching costs that prevent them from capturing all the opportunities in the liberalized market.

Figure 1 represents percentage average gains available to domestic consumers willing to switch to the best free market offer found on the PCW. As expected Scenario 1, leaving consumer's choice completely unrestricted, involves the highest gains: the latter range from about 12% of the total bill for higher consumption classes to 21% for consumers in the lowest consumption segment, 40€ per year. Looking at different peak/off peak consumption patterns (A,B and C), higher volumes on the night slot guarantee the highest savings but, overall, differences are negligible. Scenarios 2, that restricts consumer choice to one-rate tariffs, displays reduced gains for consumers with unbalanced consumption, while gains remain unchanged in Scenario 2C. This is due to the fact that in this

[5] Results for consumers with 1.5 kW are in general very similar to those reported. Consumers with a lower installed power tend to have slightly higher gains, especially if belonging to the first consumption class. A more comprehensive analysis of the gains and losses in the free market is in Airoldi and Polo (2017).

[6] We discuss here the average bill computed assigning the same weight to all 75 offers available. Results using as weights the market share of the operators give very similar results. The bill is computed according to a 30/70 consumption profile during peak and off-peak hours.

[7] A tariff offered on the free market can differ from the regulated tariff nearly exclusively for the "energy share", which represents less than half of the final bill. Then, it is argued, the annual bill could not differ that much across offers.

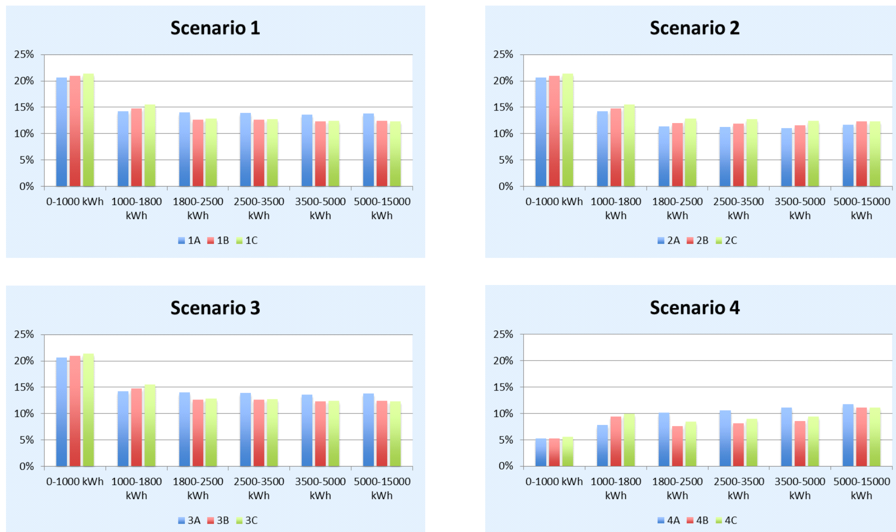

Figure 1: Gains from switching

case the cheapest tariff in the free market is a single rate-one.[8] Scenario 3, where tariffs selected involve a fixed 12 month energy price, displays the same savings, as in Scenario 1, since cheapest tariffs involve a 12 month fixed energy price. Finally, when restricting tariff choice to those options that involve a payment through postal slip (Scenario 4), gains are reduced yet significant. Interestingly, reduced gains mostly depend on the additional cost of paying with postal slips [9].

Our third relevant finding is referred to rip-offs. Potential losses for consumers moving to the free market may be quite significant.[10] This means that

[8] In Scenario 2, it is interesting to note that greatest gains are mostly obtained in the type C scenario, i.e. when there is higher daily consumption. This may happen for two reasons. First, one rate tariff bills are independent from the allocation of daily consumption, while on the other hand, the regulated regime tariff is a two-rate tariff, hence relatively more expensive in type C scenarios. Secondly, some operators that offer both single-rate and two-rate tariffs may instruct the PCW to show only one of the two depending on the consumption pattern provided by the user. Hence, some single rate offers available to consumers may be left out by the PCW. This implies that we may be underestimating the gains available in scenarios 2A and 2B.

[9] In other words, prices net of "payment costs" are not far from the lowest offers. Once taking into account additional costs associated with postal slips payment (around 12 euros plus taxes), the price paid by consumers, especially with small consumption volumes, becomes close to the regulated tariff

[10] We have carefully analyzed the best and worst contracts to exclude the possibility that the more costly offers include additional services that instead are not provided in the cheaper ones.

consumers choosing the "wrong" tariff in the free market may end up spending significantly more than with the regulated tariff. Indeed, rip-offs are not an accident in the retail Italian market, as the evidence on the average bill has shown. About 40% of the available contracts are more expensive than the regulated one, with the worst ones implying substantial losses.

Figure 2 reports our findings for the first quarter of 2017. The first consumption class displays the highest possible losses - up to 32% in Scenario 1. Potential losses for other classes remain below 15% in Scenario 1A and 1B, while they are quite higher in Scenario 1C, where they are between 18% and 21%. This can be explained by a mismatch of the tariff type in the most expensive offer (which is a two-rate tariff) with the distribution of consumption during the day in Scenario C. In fact, when restricting choice to single rate tariffs, potential losses in Scenario C are reduced and more in line with the other time profiles (with the exception of the first consumption class, between 12% and 15%). Moving to tariffs with an energy price fixed for 12 month (Scenario 3) does not change much the available losses.[11] Finally, for small consumers that want to keep paying their bill through postal payment slip potential losses may be even higher than in the unconstrained case, reaching 39%.for those with the smallest consumption (a 7 point increase with respect to Scenario 1). This is due to the fact that, in the former scenarios, we assume consumers choose the cheapest payment method available, whereas in Scenario 4 they are willing to pay an additional cost to pay by postal slip.[12] This entails a fixed cost, that implies higher percentage increases for those belonging to lower consumption classes.

These significant potential losses and the average bill of the *available* contracts to be higher than the regulated one may remind the results of the survey of the regulator on households in the free market, that reported an average bill on *subscribed* contracts to be more costly than the regulated one. There are, however, some differences between these two pieces of evidence. Our analysis is based on the offers on the PCW and allows to identify *retailers' commercial strategies towards new clients*. We cannot extend our analysis to the *contracts* that *households choose or have chosen* in the past, since the regulator does not release these latter data. Our evidence shows that indeed retailers find it convenient to offer in the free market a significant number of contracts more costly than the default one. The research question is then why it may be convenient to choose such strategies.

Taking the cheaper and more costly 10% of contracts we found that there is no systematic difference between the two groups in terms of contractual clauses.

[11] There is a difference of up to 2 percentage points in scenarios 3A and 3B, where the unconstrained worst tariff are those with a price fixed for 24 months, and no change in Scenario 3C.

[12] Note that in Scenario 1 we consider all the tariffs at their lowest available price. On the other hand, in Scenario 4 we exclude those tariffs that do not allow postal payment and, for those offers who have such option, we add the applicable surcharge.

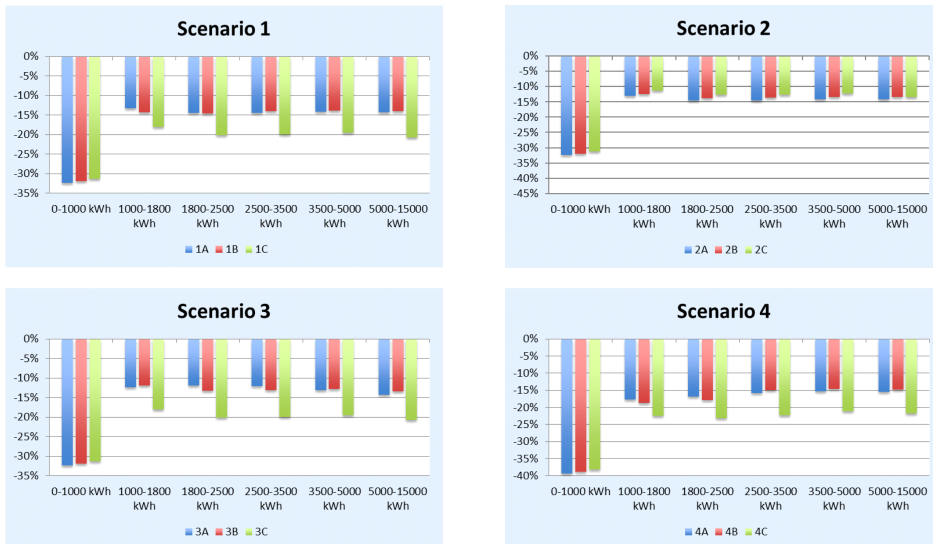

Figure 2: Losses from switching

The survey of the regulator, instead, is based on new contracts subscribed and on contracts chosen in the past and still running, whose price may have been updated over time by the retailer. The results show that the actual contracts in the free market are on average more costly than the default one. Two complementary explanation may account of this evidence. Some new customers that move to the free market may mistakenly pick up one of the costly contracts that we show are indeed extensively offered. Furthermore, it may be that some prices that were originally convenient when the household moved to the free market have been unilaterally increased by the retailer, with the customer sticking on the old contract due to switching costs.[13]

3 The model

The evidence discussed in the previous section shows that the retail electricity market in Italy is characterized by low participation, price dispersion and contracts proposed to new clients that are on average more costly, in some cases are cheaper but in others are more expensive than the regulated one.

[13]One additional reason why the Regulator finds the average bill in the free market to be higher than the regulated contract is due to additional services or charges that are included in the free market bills. Indeed, to a deeper inspection the survey does not clean the data by excluding items in the free market contracts that are not in the regulated one.

Price dispersion emerges in equilibrium under two different approaches. If firms do not observe heterogeneous consumers' types, second degree price discrimination requires to offer a menu of contracts with different prices. With this perspective asymmetric information on the side of firms is a central feature. A second approach that generates price dispersion, instead, stresses informational frictions on the side of consumers, that have to search to observe prices and product characteristics. We argue that the evidence on retail electricity markets suggests as a central theme imperfect consumers' information on market offers. Hence, our modeling choice follows a consumers' search rather than a price discrimination approach.[14]

Price dispersion and low consumers' engagement may be obtained in a model of sequential search as, for instance, Janssen et al. (2005). However, firms offering prices higher than the regulated tariff would not. Indeed, if consumers have a clear understanding of the regulated price they are paying no firm would propose a contract more expensive than that, being sure that it would be rejected even if sampled.

Transaction costs and behavioral traits may help finding an explanation of these facts. However, we argue that most of the standard explanation are unfit for this purpose. Switching costs could explain why consumers already in the free market do not react when their electricity provider unilaterally raises the price. This ingredient may explain, for instance, the evidence found by the regulator of an average bill in the free market that is higher than the regulated one. However, switching cost could not explain why retailers propose to customers served according to the default price a *new* contract that is more costly. Indeed, if moving to the free market entails switching costs retailers should try to attract new customers with low rather than high introductory prices. Alternatively, a too low regulated price might explain low participation but hardly generates equilibrium offers that are cheaper than that.

We analyze instead the case when some consumers have a biased perception of their current regulated price. This perception bias affects the comparison and may lead to accept contracts more costly than the regulated one. It comes out that this extension leads to a generalization of the Janssen et al. (2005) model. We obtain a wider and richer set of equilibria, some of which display prices higher than the regulated tariff and unconventional comparative statics properties.

3.1 The set-up

In this section we present the set-up and the main assumptions, considering the agents, payoffs, information and timing. We analyze a market with $h = 1, .., n$ **firms** offering a homogeneous product (electricity service) with identical

[14] Evidence from the PCW shows also that firms offer several and different contracts. Hence, price discrimination is an additional ingredient in the retail electricity market. There are very few papers in the literature that combine consumers' search and multiproduct retailers that price discriminate (Fabra and Montero, 2017). This interesting extension is left to future research.

and constant marginal cost that we normalize to zero. Hence, a firm's profits correspond to its revenues.

The demand side is populated by a total mass of **consumers** that we normalize to 1. Consumers have a rigid (unit) demand for electricity and utility $v - p$ (gross of any search cost) from the electricity service provided at price p. Consumers are all initially served according to a regulated tariff p_R, the only available contract pre-liberalization. Once the retail market opens they can decide to remain with the running contract or search for alternative offers on the free market. Consumers are heterogeneous under two dimensions: the *perception* regarding the price paid according to the initial contract and the *search cost* for additional price quotes.

More precisely, consumers receive different **signals** p_0^i on the **current regulated price** they are paying for the electricity service (we use superscript ^{i} to refer to this first dimension of heterogeneity). Consumers may be of a "low" type (^{L} superscript), with a downward perception of the price paid, "unbiased" (^{U} superscript) or of a "high" type (^{H} superscript), when the perceived price paid according to the regulated contract is higher than the true one. Hence, $p_0^i \in \{p_0^L, p_0^U, p_0^H\}$, where $p_0^L = p_R - k$, $p_0^U = p_R$ and $p_0^H = p_R + k$, with $k \in (0, p_R)$. When consumers sample a new price, they perceive it correctly.[15]

Secondly, consumers have a different **search cost** c^j for price quotes on the free market (superscript ^{j} labels types under this second dimension). Costs may be zero, corresponding to what we call a "shopper" type (superscript ^{S}), or they may be $c > 0$, identifying a "non-shopper" type (superscript ^{NS}). Hence, $c^S = 0$ and $c^{NS} = c$.

The two dimensions of heterogeneity are combined as follows. There is a percentage μ of shoppers and the residual share $1 - \mu$ of non-shoppers. All the "shoppers" receive an unbiased signal of the price they are paying on the running regulated contract: $\Pr(S, U) = \mu$. We justify this assumption on the ground that "shoppers" are consumers that have no cost of being informed, and therefore they tend to be correctly informed also on the price they are paying on the running contract. Conversely, we assume that "non-shoppers" receive biased signals, with an equal share of "low" and "high" types: $\Pr(NS, L) =$

[15] In our setting, therefore, non-shoppers mistakenly perceive their regulated price but correctly observe a new quote. Alternatively one might assume that non-shoppers have a distorted perception of their current level of consumption. Then, if the electricity bill is composed by a fixed part and one that depends on consumption, errors in the level of expected consumption could affect the ranking of the regulated and new contracts. We believe that this explanation can only partially explain pricing dynamics in the Italian market. First of all, in Italy the fixed component of the bill mostly includes regulatory and fiscal items. The only fixed component set by free market operators, a commercialization charge, is indexed and set equal to the regulated tariff's commercialization charge in 88% of the tariffs in our sample. If we include una tantum fixed discounts, about 70% of the offers in our sample have a fixed component equal to the regulated one, including many offers more expensive than it. Hence, we take as a reference the case where the fixed component is the same in the contracts on the free market and in the regulated one. Then, the difference in the expected cost of most contracts depends on the variable component only. But if consumers compare contracts based on the same (wrong) level of consumption they would distort in the same direction all the estimates, without affecting their relative ranking.

$\Pr(NS, H) = \frac{1-\mu}{2}$. Since the regulated contract has been subscribed in the past, with the regulated price updated from time to time by the authority, consumers that have a cost to collect information quite naturally tend also to misperceive the current price paid in the electricity bill.

Consumers search sequentially with perfect recall. If they search for a first quote p_1 they sample with the same probability a price quoted by any of the n firms. Then, they can decide whether to carry on with the regulated contract, paying the (perceived) price p_0^i, accept the price p_1 or proceed with a further search and obtain a second quote p_2, and so on. Since consumers have perfect recall, we can define $P_t^i = \{p_0^i, p_1, ..., p_t\}$, $i = L, H, U$, as the set of $t+1$ prices available after t searches, and $\underline{p}_t^i = \min P_t^i$ the lowest price among those available.[16] Summing up, after t searches the relevant choice for a consumer of type (S, U), (NS, L) and (NS, H) is between paying $\underline{p}_t^i$, $i = L, H, U$, and searching for a further quote, incurring a search cost c^j, $j = S, NS$.

In order to guarantee some positive participation from non-shoppers in the free market, we introduce the following:

Assumption 1: $p_R + k > c$.

The **timing** of the game is as follows.

- At **stage** 0 all the consumers are served according to the regulated contract. Nature draws consumers' types with $\Pr(S, U) = \mu$ and $\Pr(NS, L) = \Pr(NS, H) = \frac{1-\mu}{2}$. Consumers observe their type[17] while firms know the distribution of types but not the individual realizations.

- At **stage** 1 firms $h = 1, .., n$ simultaneously choose a price probability distribution $f_h(p)$ over a given support, and each firm h draws a price according to $f_h(p)$.

- At **stage** 2 consumers of type (S, U), (NS, L) and (NS, H) decide to carry on with the running contract or to search sequentially starting from $t = 1, 2, ..$ given their search costs c^j, $j = S, NS$ and the signal on the current price paid p_0^i, $i = L, U, H$, the firms' pricing strategies $F_h(p)$ and the set of available prices P_t^i.

We restrict our analysis to the case of symmetric mixed strategies, an assumption that is usually adopted in the search literature.

[16] Notice that since P_t^i includes the initial signal p_0^i on the regulated price, the set of available prices differs across types $i = L, H$.

[17] We define for convenience non-shopper as either high or low type. Assuming that a non-shopper of type $i = H, L$ observes her type means that she knows her search cost c and the initial price p_0^i she thinks to pay, while she does not know the true level of the regulated price, p_R. Hence, obviously, being, for instance, a "high" type does not mean that the consumer is aware of perceiving a regulated price higher than the true one.

4 Equilibrium analysis

Our set up corresponds to a screening game with two-dimensional adverse selection and we look for a symmetric Perfect Bayesian Equilibrium with firms playing a pricing strategy $F^*(p)$ given consumers' equilibrium strategies and consumers optimally participating, searching and choosing a price given firms' equilibrium strategies. Firms' beliefs on consumers' types are correct in equilibrium. We start from consumers' choices on search and purchase at stage 2 and then move backward to firms' pricing strategies at stage 1.

4.1 Consumer Search

Let us consider the search behavior of an individual consumer of different type for given symmetric pricing strategies of the firms, $f(p)$. Note that $f(p)$ may be continuous, discrete or mixed depending on whether there are some prices $p_l \in P_l$ at which the price density distribution has an atom. From now on, we will refer to $\hat{f}(p)$ as the continuous part and to $\Pr(p = p_l)$ as the atoms of the price density distribution (if there are some). This general formulation covers a number of equilibrium scenarios.

Shoppers find it weakly optimal to search n times, sampling all the prices, for any pricing strategy $f_h(p)$ adopted by the firms. Indeed, since searching is not costly, their utility is not reduced by such activity whereas it may happen that, along the equilibrium path or out of it, they find an offer lower than the initial price p_R. Non-shoppers, instead, incur a cost c for each search they run. Then, at a given stage they will further search if the price they expect to sample is sufficiently lower than the best available alternative to cover the search cost. The search strategy of a non-shoppers is based on a relevant threshold, as stated in the following Lemma. We move this and all the other proofs in Appendix II.

Lemma 1 **(Reservation price)**: *Given a symmetric mixed strategy $f(p)$ a non-shopper is characterized by a unique reservation price r defined by*

$$\int_{\underline{p}}^{r} (r - p)\hat{f}(p)dp + \sum_{p_l \in P_l} (r - p_l) \Pr(p = p_l | p \leq r) - c = 0, \tag{1}$$

where $\underline{p}$ is the lower bound of the continuous distribution $\hat{f}(p)$ and $p_l \in P_l$ are those prices (if any) where the price distribution has an atom.

The reservation price (1) identifies an available price level r that makes the non-shopper with search costs c indifferent between accepting r or running a further search, given the expectation on the prices that will be quoted through search based on the symmetric mixed strategy $f(p)$. It depends on the price distribution $f(p)$, that affects the expected price that may be sampled by searching, and the search cost c incurred, whereas it does not depend on the initial signal p_0^i. In other words, non-shoppers of high or low type, having the same search cost c, share the same reservation price r when they consider the opportunities offered by searching in the free market. Moreover, in a symmetric (mixed) strategy configuration the price distribution $f(p)$ that generates the prices of the residual, unsampled firms in search $t+1$ is the same no matter how many searches have already been carried out nor who are the firms already sampled. To sum up, the reservation price r is independent from the number of searches t and the signal on the regulated price paid p_0^i.

Corollary 1 establishes the optimal search behavior of non-shoppers after $t \geq 0$ searches:

Corollary 1 **(Optimal stopping rule of non-shoppers)**: *Non-shoppers $i = L, H$ after $t \geq 0$ searches*:

- *Search a new offer if the most convenient price among the $t+1$ available is higher than the reservation price: $\underline{p}_t^i > r$, unless all the prices have already been sampled, i.e. if $t = n$.*
- *Stop and purchase at the lowest available price $\underline{p}_t^i$ if $\underline{p}_t^i < r$.*
- *If $\underline{p}_t^i = r$ and $t = 0$ non-shoppers search with positive probability, when $t > 0$ non-shoppers stop searching and purchase;*[18]

Notice that the stopping rule implies that when a consumer searches and purchases, it selects the latest sampled price. The stopping rule holds after any number of searches $t \geq 0$. At $t = 0$ a non-shopper $i = L, H$ decides to carry out a first search and enters the free market if $U_0^{iNS} \leq U_1^{iNS}$, that is:

$$\int_{\underline{p}}^{p_0^i} (p_0^i - p)\hat{f}(p)dp + \sum_{p_l \in P_l} (p_0^i - p_l) \Pr(p = p_l | p \leq p_0^i) - c > 0, \tag{2}$$

where we exploit the fact that $\underline{p}_0^i = p_0^i$ by definition. Hence, the stopping rule implies that a non-shopper $i = L, H$ will run a first search with probability 1 if $p_0^i > r$. In case of indifference, when $p_0^i = r$, according to the stopping rule non-shoppers search with a positive probability. Let θ_1^L, θ_1^H be, respectively, the probability of running a first search by the low and high non-shoppers. In the following Lemma we figure out the entry behavior of non-shoppers according to the stopping rule.

Lemma 2 **(Equilibrium entry decisions)**: *In any PBE it cannot be that $\theta_1^H < \theta_1^L$.*

[18] This condition is required to ensure that a mixed strategy equilibrium $f^*(p)$ exists. Since non-shoppers are indifferent to a further search when $p_t^i = r$ not searching after the first sample is optimal.

Given Lemma 2 there are a number of combinations of θ_1^L and θ_1^H that may old in equilibrium. If (2) holds for both types, all non-shoppers run a first search, with $\theta_1^L = \theta_1^H = 1$. Alternatively, the low type is indifferent while the high type strictly prefers to run a first search. According to the stopping rule, the low type randomizes while the high type certainly enters, leading to $0 < \theta_1^L \leq \theta_1^H = 1$. Third, if (2) holds for the high type while the low type strictly prefers not to enter, then $0 = \theta_1^L < \theta_1^H = 1$. If (2) holds as an equality for the high type and does not hold for the low type, the former randomizes, that is $0 = \theta_1^L < \theta_1^H \leq 1$. Finally, if (2) does not hold for either type, $0 = \theta_1^L = \theta_1^H$. [19]

We can define the average participation rates of non-shoppers as

$$\theta_1 = \frac{\theta_1^H + \theta_1^L}{2}. \tag{3}$$

Since $0 < \theta_1^L \leq \theta_1^H = 1$ or $0 \leq \theta_1^L < \theta_1^H \leq 1$, any value of $\theta_1 \in [0, 1]$ is uniquely associated to a pair of individual participations (θ_1^L, θ_1^H). [20] Hence, with no ambiguity we can refer to the entry behavior of non-shoppers using θ_1.

4.2 Pricing strategies

Having analyzed the search behavior of the consumers at stage 2 for given symmetric price density distribution $f(p)$ we move back to stage 1 and analyze firms' pricing strategies. The superscript * refers to the equilibrium price density distributions. We start with two results implied by consumers' search and purchase behavior discussed in Section 4.1.

Corollary 2 **(Upper bound of the equilibrium price distribution I)**: *Given the optimal purchase behavior of consumers, any mixed strategy such that the upper bound $\bar{p} > p_R + k$ cannot be optimal.*

This result is a direct consequence of Corollary 1, second bullet. Given that consumers receive a signal p_0^i of the regulated price at time 0, they will never purchase an offer at a price $p > p_0^i$. Hence, any offer with price above $p_R + k$ will yield null profits to firms. We will see later on in this section that firms can instead gain positive profits by playing prices belonging to the equilibrium price distribution. Thus, playing prices greater than $p_R + k$ with positive probability cannot be part of an equilibrium. In the rest of this section, when we refer to the optimal price density distribution we allude to those distributions such that $\bar{p} \leq p_R + k$.

[19] We shall see in the next section on pricing strategies that this latter configuration is incompatible with a PBE.

[20] For instance, if $\theta_1 \in \left[\frac{1}{2}, 1\right)$ then $\theta_1^H = 1$ and $\theta_1^L \in (0, 1)$ while $\theta_1 \in \left(0, \frac{1}{2}\right)$ implies $\theta_1^H \in (0, 1)$ and $\theta_1^L = 0$.

Lemma 3 (Upper bound of the equilibrium price distribution II): *Given the participation and search optimal behavior of non-shoppers, the upper bound of the optimal price density distribution $f(p)$ is $\bar{p} = r$ and non-shoppers search at most once.*

5 Equilibria

Our first result shows that no equilibrium exists with all non-shoppers staying out of the market, that is $\theta_1 = 0$.

Lemma 4: *If a PBE exists, at least some H-type non shoppers participate, that is $\theta_1 > 0$.*

Given Lemma 4, shoppers and (some) non shoppers enter the market in equilibrium. In this case no pure strategy equilibrium exists. In fact, in this case the profit functions are not quasi-concave: competing for shoppers entails Bertrand competition, but a price equal to the marginal cost is not an equilibrium since any firm would obtain positive profits raising the price up to the reservation price of non-shoppers and serving a positive fraction of them. This latter price, however, would not be an equilibrium either, since by slightly undercutting the rivals a firm would secure all the shoppers. Then, firms mix on a continuous support and, in some cases, play a high price with a positive probability, with all shoppers and at least some high-type non-shoppers participating. [21]

The timing of the game implies that consumers choose optimally their participation rate θ_1^L and θ_1^H and reservation price r given the mixed strategy $F(p)$ chosen by firms, and that these latter choose the optimal mixed strategy anticipating the optimal choice of consumers. We first derive the equilibrium conditions for given θ_1 and then we find the optimal participation rate. Given non-shoppers participation θ_1, the optimal mixed strategy $f(p)$ determines the reservation price r according to (1). The profits of a firm when the others follow a symmetric mixed strategy $F(p; r, \theta_1)$ are:

$$\pi_h(p, F(.)) = \begin{cases} p\left[\frac{(1-\mu)\theta_1}{n} + \mu(1-F(.))^{n-1}\right] & \text{if } p \in [0, min\{p_R, r\}] \\ p\frac{(1-\mu)\theta_1}{n} & \text{if } p \in (min\{p_R, r\}, r] \\ 0 & \text{if } p > r \end{cases} \tag{4}$$

[21] We can, instead, exclude pure strategies price equilibria in which only high-type non shoppers participate. In this case, the (candidate) equilibrium price p should be sufficiently low to induce high-type non shoppers to search, that is $p = p_R + k - c - \varepsilon$. But since non-shoppers search according to the (candidate) equilibrium price, any firm would have an incentive to deviate and set $p = p_R + k - \varepsilon$, knowing that, if randomly sampled, its price would be accepted.

where the first expression corresponds to the profits when price p is below the price already available to the shoppers, i.e. if $p \leq p_R$. In this case the firm sells to all shoppers if it posts the lowest price in the market (which happens with probability $(1 - F(.))^{n-1}$) and to a fraction $1/n$ of non-shoppers that has entered the market and sampled its price, accepting it since it is lower than r. Given Lemma 3, if $r > p_R$ the second expression describes the expected profits for $r \geq p > p_R$. In this case the firm sells only to a fraction $1/n$ of the active non-shoppers for a price not higher than their reservation price. Finally, for a price higher than r the firm sells nothing.

Let us introduce the following threshold:

$$\overline{\mu}(r, \theta_1) = \frac{r - p_R}{r - p_R + np_R/\theta_1}$$

that identifies the values of μ such that a mixed strategy $F(p)$ exists for given θ_1 and r. In the following Lemma we characterize the optimal mixed strategy given the level of participation of non-shoppers, θ_1, identified by a $\widetilde{}$.

Lemma 5 **(Characterization of the optimal mixed strategy for given** θ_1**)**: *Given* θ_1 *and parameters* (μ, n, c, k, p_R)*, if* $\mu > \overline{\mu}(r, \theta_1)$ *the optimal symmetric mixed strategy is*

$$\widetilde{F}(p; r, \theta_1) = \begin{cases} 1 - \left[\frac{\theta_1(r-p)}{nbp}\right]^{\frac{1}{n-1}} & \textit{for} \quad p \in \left[\underline{p}, \min\{p_R, r\}\right] \\ 1 - \left[\frac{\theta_1(r-p_R)}{nbp_R}\right]^{\frac{1}{n-1}} & \textit{for} \quad p \in (\min\{p_R, r\}, r) \\ 1 & \textit{for } p \geq r \end{cases} \tag{5}$$

where $b = \frac{\mu}{1-\mu}$. *The support of the optimal mixed strategy is*:

$$\left[\frac{(1-\mu)\theta_1 r}{\mu n + (1-\mu)\theta_1}, \min\{p_R, r\}\right] \cup r,$$

where the optimal reservation price is defined by the condition

$$r = \int_{\underline{p}}^{p_R} p \frac{\widetilde{f}(p; r, \theta_1)}{\widetilde{F}(p_R; r, \theta_1)} dp + \frac{c}{\widetilde{F}(p_R; r, \theta_1)} = E(p; r, \theta_1) + c. \tag{6}$$

The intuition behind this result is the following: since in the market there are shoppers, who sample all price and buy the best deal if it is lower than p_R, and the mass θ_1 of non-shoppers, who search at most once accepting the first price (not higher than r) sampled, firms trade-off these two components of revenues. If any firm would set a price $p < p_R$ with a strictly positive probability, that is adopting a density distribution with an atom below p_R, than all other firms would gain by playing with a positive probability a price slightly below p, stealing from the first firm all the shoppers. Then, firms randomize their prices adopting a price density distribution with no atoms in the interval $\left[\underline{p}, \min\{p_R, r\}\right]$. The mixed strategy balances the benefits coming from winning more shoppers by playing with a higher probability low prices with the rents they can extract from non-shoppers by putting more weight on higher prices. Since active non-shoppers accept any price not higher than the reservation price, if $p_R < r$ any price $p \in (p_R, r]$ would be accepted only by the (active) non-shoppers. Then the firms choose with a positive probability $1 - \widetilde{F}(p_R)$ the highest acceptable price for non-shoppers, that is the reservation price r. In this latter case, therefore, the mixed strategy entails an atom at r.

Lemma 5 shows the optimal mixed strategy and reservation price for given θ_1. Likewise, equation (6) claims that there is a relationship between the reservation price r and the participation rate of non-shoppers θ_1 given the optimal mixed strategy of the firms (for given θ_1). To show this we retrieve from the mixed strategy (5) the price:

$$p = \frac{r}{1 + \frac{bn}{\theta_1}(1 - \widetilde{F}(p; r, \theta_1))^{n-1}} \quad \text{for} \quad p \leq p_R. \tag{7}$$

Let us set $\widetilde{y}(p; r, \theta_1) = 1 - \widetilde{F}(p; r, \theta_1)$. When $p_R < r$, following the pattern of $\widetilde{F}(p)$, $\widetilde{y}(p)$ is decreasing in $p \in \left[\underline{p}, p_R\right]$, constant in the interval $p \in (p_R, r)$ and jumps down to zero at the atom $p = r$, whereas it is always decreasing in $p \in \left[\underline{p}, r\right]$ when $r \leq p_R$. Then, we can write $E(p) = \int_{\widetilde{y}(p_R)}^{1} p dy + r\widetilde{y}(p_R)$ and, substituting p with condition (7) we obtain:

$$\widetilde{E}(p; r, \theta_1) = r[\int_{\widetilde{y}(p_R; r, \theta_1)}^{1} \frac{dy}{1 + \frac{bn}{\theta_1}\widetilde{y}\,(p; r, \theta_1)^{n-1}} + \widetilde{y}(p_R; r, \theta_1)] \tag{8}$$

where, again, the decoration $\widetilde{}$ stands for the variables expressed at their optimal level for given θ_1. Then, (6) given (8) can be written as

$$G(r, \theta_1) := r * \widetilde{\Phi}(r, \theta_1) = c \tag{9}$$

where[22]

$$\widetilde{\Phi}(r, \theta_1) = \begin{cases} 1 - \int_0^1 \frac{dy}{1 + \frac{bn}{\theta_1}\widetilde{y}(p; r, \theta_1)^{n-1}} & \text{for} \quad r \leq p_R \\ 1 - \widetilde{y}(p_R;\, r, \theta_1) - \int_{\widetilde{y}(p_R; r, \theta_1)}^{1} \frac{dy}{1 + \frac{bn}{\theta_1}\widetilde{y}(p; r, \theta_1)^{n-1}} & \text{for} \quad r > p_R \end{cases} \tag{10}$$

The condition $G(r, \theta_1) = c$ defines contours $\widetilde{r}(\theta_1)$ in the (r, θ_1) space, that we interpret as the combinations of the participation rate θ_1 and the *optimal* reservation price associated with the *optimal* mixed strategy $\widetilde{F}(.)$.

According to (9), as shown in Appendix III, when $r \leq p_R$ the locus $\widetilde{r}(\theta_1)$ is strictly increasing in θ_1. In this case, that is the same as in Janssen et al.

[22] Note that the expression on the first line corresponds to Janssen et al. (2005) in case of partial participation. Since in their model non-shoppers do not have an outside option lower than shoppers, as instead is the case in our model for high types, these author have not to consider the case of a mixed strategy with an atom at the upper bound, that gives the second expression.

(2005), the mixed strategy has no atom and an increase in the participation of non-shoppers moves the price distribution to the right, increasing $E(p)$ and r. When instead $r > p_R$ the support has a gap in the interval (p_R, r) and the mixed strategy has an atom at $p = r$ that is played with probability $1 - \widetilde{F}(p_R)$. In this case the impact of an increase in the level of participation θ_1 on the reservation price derives from two opposite effects. On the one hand an increase in the number of non-shoppers pushes the price distribution to the right with an increase in $\widetilde{E}(p)$ and r, as in the previous case. On the other hand, the probability of the atom and the lower bound of the mixed strategy $\underline{p}$ increase and, in order to keep firms indifferent among the prices in the support, the upper bound r decreases, making $\widetilde{E}(p)$ falling as well. As a combination of these two forces, the locus $\widetilde{r}(\theta_1)$ is still initially increasing in θ_1 for $r > p_R$ and then it is backward banning and becomes decreasing in θ_1, when the second component prevails. We define θ_1^D the value of the participation rate at which the locus $\widetilde{r}(\theta_1)$ changes slope, with $\widetilde{r}(\theta_1^D) > p_R$.

From the previous discussion it follows that, for a given $\theta_1 < \theta_1^D$, the reservation price has two values, $\widetilde{r}_1(\theta_1) < \widetilde{r}_2(\theta_1)$, that are consistent with the equilibrium mixed strategies and search behavior. In other words, for a given θ_1 there are two candidate equilibrium configurations that satisfy the indifference condition of the mixed strategy and the optimal stopping rule based on the reservation price, one associated with a low and the other with a high reservation price r and expected price $\widetilde{E}(p)$.

Turning to the optimal decision to search for the first time, we can close the model and verify whether both values $\widetilde{r}_1(\theta_1)$ and $\widetilde{r}_2(\theta_1)$ or just one of them are part of an equilibrium in the overall game. We know from Lemma 1 and 2 that non-shoppers of type $i = H, L$ fully participate if $p_0^i > r$, randomize choosing a $\theta_1^i \in [0,1]$ if $p_0^i = r$ and do not run any search if $p_0^i < r$. Moreover, if only H types participate $\theta_1 \in \left(0, \frac{1}{2}\right]$ whereas if also L types search then $\theta_1 \in \left(\frac{1}{2}, 1\right]$. Hence, the optimal participation of non-shoppers for given reservation price r is summarized by the following:

$$\widetilde{\theta}_1(r) = \begin{cases} 0 & \text{if } r > p_{R+k} \\ \left(0, \frac{1}{2}\right) & \text{if } r = p_R + k \\ \frac{1}{2} & \text{if } r \in (p_R - k, p_R + k) \\ \left(\frac{1}{2}, 1\right) & \text{if } r = p_R - k \\ 1 & \text{if } r < p_R - k \end{cases} . \tag{11}$$

Then, a PBE in the game is a triple

$$\left\{F^*(p) = \widetilde{F}(p; r^*, \theta_1^*),\ r^* = \widetilde{r}(\theta_1^* F^*),\ \theta_1^* = \widetilde{\theta}_1(r^*, F^*)\right\}$$

such that the firms play the optimal mixed strategy (5) given the optimal reservation price and participation rate, shoppers always search, non-shoppers optimally search with probability (11) and follow the stopping rule according to the reservation price (9) given the optimal mixed strategy. We can conveniently represent the equilibria in this game in the space (θ_1, r) through the points of intersection between the locus $\widetilde{r}(\theta_1)$ and the schedule $\widetilde{\theta}_1(r)$. In the following Propositions we characterize the different PBE.

Proposition 1. *If $\widetilde{r}_2(\frac{1}{2}) > p_R + k$ the symmetric PBE is unique. Moreover:*

1.1: *if $\widetilde{r}_1(1) \leq p_R - k$ then both types fully participate: $\theta_1^{1.1} = 1$; the equilibrium mixed strategy has no atom.*

1.2: *if $\widetilde{r}_1(\frac{1}{2}) \leq p_R - k < \widetilde{r}_1(1) < p_R + k$ then high types fully participate and low types randomize: $\theta_1^{1.2} \in \left[\frac{1}{2}, 1\right)$ where $\widetilde{r}_1(\theta_1^{1.2}) = p_R - k$; the equilibrium mixed strategy has no atom.*

1.3: *if $\mu \geq \overline{\mu}(\frac{1}{2})$ and $p_R - k < \widetilde{r}_1(\frac{1}{2}) < p_R + k$ then high types fully participate and low types stay out: $\theta_1^{1.3} = \frac{1}{2}$; the equilibrium mixed strategy has an atom if $\widetilde{r}_1(\theta_1^{1.3}) > p_R$.*

1.4: *if $\mu \geq \overline{\mu}(\theta_1^{1.4})$ and $p_R + k \in \left(c, r_1(\frac{1}{2})\right]$ then high types randomize and low types stay out: $\theta_1^{1.4} \in \left(0, \frac{1}{2}\right)$ where $\widetilde{r}_1(\theta_1^{1.4}) = p_R + k$; the mixed strategy has an atom at $\widetilde{r}_1(\theta_1^{1.4})$.*

Proposition 1 illustrates four classes of unique symmetric PBE in mixed strategies characterized by decreasing level of participation of non-shoppers and increasing maximum price posted. Figure 3, panels 1-4 (obtained by simulating the model with Matlab), where the points $1.1-1.4$ correspond to the equilibrium configurations in Proposition 1, shows the corresponding cases. It can be noticed that when the noise of the signal is small ($k = 1$) all shoppers participate, whereas, a more noisy signal, *coeteris paribus*, reduces the participation rate of low types, that now have a more optimistic perception of the regulated contract currently running and are less willing to search. [23] We present a detailed comparative statics analysis in the next section.

We move now to a region where multiple equilibria exist.

Proposition 2. *If $\theta_1^D \geq \frac{1}{2}$, $\widetilde{r}_2(\frac{1}{2}) < p_R + k$ and $p_R - k \in \left(\widetilde{r}_1(\frac{1}{2}), \widetilde{r}_1(\min\left\{\theta_1^D, 1\right\}\right]$ there exist three symmetric PBE:*

2.1: *if $\mu \geq \overline{\mu}(\theta_1^{2.1})$, high types randomize and $\theta_1^{2.1} \in \left(0, \frac{1}{2}\right)$ such that $\widetilde{r}_2(\theta_1^{2.1}) = p_R + k$; the equilibrium mixed strategy has an atom at $\widetilde{r}_2(\theta_1^{2.1})$.*

2.2: *if $\mu \geq \overline{\mu}(\frac{1}{2})$, high types fully participate, $\theta_1^{2.2} = \frac{1}{2}$ with $\widetilde{r}_2(\frac{1}{2}) < p_R + k$; the equilibrium mixed strategy has an atom at $\widetilde{r}_2(\frac{1}{2})$.*

2.3: *high types fully participate and low types randomize (fully participate), $\theta_1^{2,3} \in \left(\frac{1}{2}, 1\right)$ ($\theta_1^{2,3} = 1$) such that $\widetilde{r}_1(\theta_1^{2.3}) = p_R - k$ ($\widetilde{r}_1(\theta_1^{2.3}) < p_R - k$); the equilibrium mixed strategy has no atom.*

[23] These two equilibtia correspond to those in Janssen et al. (2005).

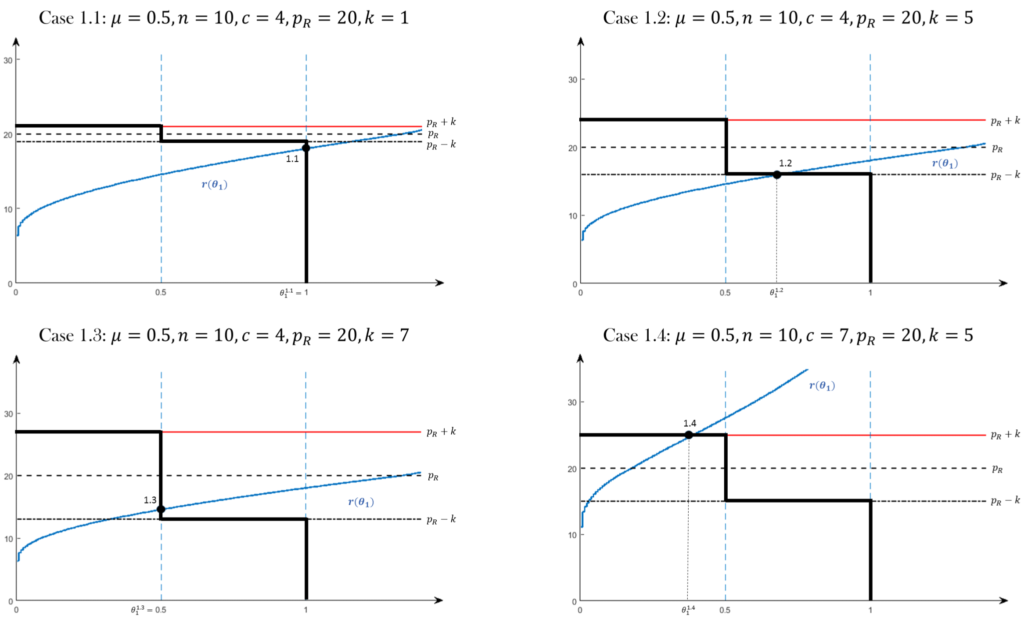

Figure 3: Equilibria, Proposition 1

If $\widetilde{r}_2(\frac{1}{2}) = p_R + k$, $\theta_1^D \geq \frac{1}{2}$ *and* $p_R - k \in \left(\widetilde{r}_1(\frac{1}{2}), \widetilde{r}_1(\min\left\{\theta_1^D, 1\right\}\right]$ *then equilibria* 2.1 *and* 2.2 *coincide. If* $\widetilde{r}_2(\frac{1}{2}) \leq p_R + k$, $\theta_1^D \geq \frac{1}{2}$ *and* $p_R - k < \widetilde{r}_1(\frac{1}{2})$ *equilibrium* 2.3 *is such that* $\theta_1^{2.3} = \frac{1}{2}$*and* $\widetilde{r}_1(\theta_1^{2.3}) = \widetilde{r}_1(\frac{1}{2})$.

A ranking in terms of profits among the three equilibria cannot be established in general. As it is evident from Figure 4, first panel, moving from equilibrium 2.1 to 2.2 and then 2.3 the participation rate of non-shoppers raises ($\theta_1^{2.1} < \theta_1^{2.2} < \theta_1^{2.3}$), and therefore the expected sales of the firms, increase. At the same time, since $\widetilde{r}(\theta_1) = \widetilde{E}(p; \theta_1) + c$ the expected price falls since $\widetilde{r}_2(\theta_1^{2.1}) > \widetilde{r}_2(\theta_1^{2.2}) > \widetilde{r}_1(\theta_1^{2.3})$. Hence, there is an opposite pattern of the extensive and intensive margin across the three equilibria.[24]

The next Proposition shows a third class of unique PBE that completes our analysis of mixed strategy symmetric equilibria

Proposition 3: *If* $\theta_1^D < \frac{1}{2}$ *the symmetric PBE is unique. Moreover:*

3.1: *if* $\mu \geq \overline{\mu}(\theta_1^{3.1})$ *and* $\widetilde{r}(\theta_1^D) < p_R + k$ *the high types participate with probability* $\theta_1^{3.1} < \theta_1^D$ *such that* $\widetilde{r}_2(\theta_1^{3.1}) = p_R + k$ *whereas the low types stay out; the equilibrium mixed strategy has an atom at* $\widetilde{r}_2(\theta_1^{3.1})$.

[24] Only when $\theta_1^{2.3} = \frac{1}{2}$*and* $\widetilde{r}_1(\theta_1^{2.3}) = \widetilde{r}_1(\frac{1}{2})$ equilibrium 2.2 dominates equilibrium 2.3

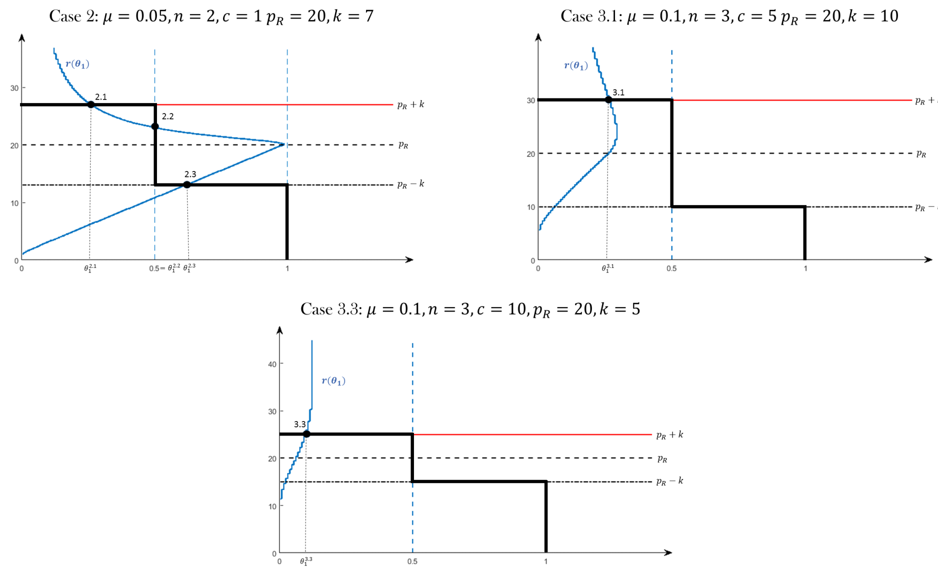

Figure 4: Equilibria, Proposition 2 and 3

3.2: *if* $\mu \geq \overline{\mu}(\theta_1^D)$ *and* $\widetilde{r}(\theta_1^D) = p_R + k$ *the high types participate with probability* $\theta_1^{3.2} = \theta_1^D$ *whereas the low types stay out; the equilibrium mixed strategy has an atom at* $\widetilde{r}(\theta_1^D)$.

3.3: *if* $\mu \geq \overline{\mu}(\theta_1^{3.3})$ *and* $\widetilde{r}(\theta_1^D) > p_R + k$ *the high types participate with probability* $\theta_1^{3.3} < \theta_1^D$ *such that* $\widetilde{r}_1(\theta_1^{3.3}) = p_R + k$*, whereas the low types stay out; the equilibrium mixed strategy has an atom at* $\widetilde{r}_1(\theta_1^{3.3})$. [25]

The different equilibria in Proposition 1-3 display price dispersion. Moreover, when only high type non-shoppers participate the maximum price posted is above the regulated price. Both features are in line with the evidence presented in Section 2 on the contracts offered in the Italian retail market and with the potential gains and losses that firms' strategies may generate. [26]

[25] We can notice that the equilibrium in 3.2 is qualitatively similar to the one in 1.4, entailing partial participation of high types and a reservation price equal to $p_R + k$. The only difference, that is irrelevant at the equilibrium point, stays in the fact that in the equilibrium 3.2 we have $\theta_1^D < \frac{1}{2}$ whereas in the equilibrium 1.4 it is $\theta_1^D > \frac{1}{2}$.

[26] Moreover, since $r^* = E^*(p; r, \theta_1^*) + c$, in the equilibria where only the high type non-shoppers participate we have $r = p_R + k$ and the expected price paid by non-shoppers, $E^*(p) = p_R + k - c$, is higher than the regulated price if $k > c$.

6 Comparative statics

In this Section we discuss the effects of changes in the perception bias and search costs of non-shoppers on the equilibrium outcome.

6.1 Changes in the perception bias k

The comparative statics of the *expected price* with respect to the perception bias k corresponds to shifting the horizontal lines at $p_R - k$ and $p_R + k$ in our figures, whereas the locus $\widetilde{r}(\theta_1)$ does not change. In the following Lemma we state the effects of a variation in the perception bias on the expected price and participation rate.

Lemma 6: *In the equilibria with* ***full participation*** *of both types (*1.1*) or high types only (*1.3 *and* 2.2*) a marginal variation in the perception bias k does not affect the expected price $E^*(p)$ and participation rate θ_1^*.*

In the equilibria with ***partial participation of low types*** *(*1.2 *and* 2.3*) the expected price $E^*(p)$ and participation rate θ_1^* are decreasing in the perception bias.*

In the equilibria with ***partial participation of high types*** *the expected price $E^*(p)$ is always increasing in the perception bias while the participation rate is increasing in k in equilibria* 1.4 *and* 3.3 and decreasing in k in equilibria 2.1 *and* 3.1.[27]

A variation in the perception bias k affects the expected price and participation rate only when in equilibrium there is partial participation. In the following table we summarize the results, specifying in the first row whether the marginal non-shopper in equilibrium is of the high or the low type.

Table 2 - Comparative statics with respect to k

$k \downarrow$	$\frac{\partial p_0^{i*}}{\partial k} > 0$ $(i = H)$	$\frac{\partial p_0^{i*}}{\partial k} < 0$ $(i = L)$
$\frac{\partial r^*}{\partial \theta_1} > 0$ $(r^* = \widetilde{r}_1(\theta_1^*))$	$E^*(p) \downarrow$, $\theta_1^* \downarrow$ eq. 1.4 and 3.3	$E^*(p) \uparrow$, $\theta_1^* \uparrow$ eq. 1.2 and 2.3
$\frac{\partial r^*}{\partial \theta_1} < 0$ $(r^* = \widetilde{r}_2(\theta_1^*))$	$E^*(p) \downarrow$, $\theta_1^* \uparrow$ eq. 2.1 and 3.1	

With partial participation a change in the perceived regulated price affects the decision to search. Whether the marginal non-shopper is of the low or high type, then, makes the difference. A fall in k makes a high type less pessimistic on the current regulated price and, given the pricing strategies of firms, less willing to search. The opposite occurs if the marginal non-shopper is of the

[27] When $p_R + k = r(\frac{1}{2})$ $(p_R - k = r(1))$ we have full participation of high (low) types but a marginal reduction (increase) in k will shift the equilibrium to the partial participation of high (low) type case.

low type.[28] The composition of active consumers between shoppers and non-shoppers then affects firms' pricing strategies. Since $E^*(p) + c = r^*(\theta_1^*)$, the equilibrium expected price, in turn, depends on whether there is a positive relationship between the participation rate and the reservation price, as it is along $\widetilde{r}_1(\theta_1)$, or a negative one ($\widetilde{r}_2(\theta_1)$).

The second row in the table corresponds to equilibria along the increasing segment of the locus, i.e. $r^*(\theta_1^*) = \widetilde{r}_1(\theta_1^*)$. In these cases the participation rate and the expected price move in the same direction: an increase in the participation of non-shoppers changes the composition of active consumers and enhances the incentives to set higher prices. The expected and the reservation price, then, increase consistently with an increase in the participation of non-shoppers. Notice that if the equilibrium entails partial participation of high types a reduction in the perception bias would reduce the expected price but also the participation of non-shoppers.

The case represented in the third row of the table, instead, displays a different feature. In equilibria 2.1 and 3.1 the reservation price is above p_R, the mixed strategy has an atom and the locus is decreasing, i.e. $r^*(\theta_1^*) = \widetilde{r}_2(\theta_1^*)$. In this case the marginal consumer is the high type. Suppose that the perception bias increases, making him more willing to participate. However an increase in θ_1 would push up too much the expected price, since the reservation price is high and the atom is played with a high probability. The high type non-shoppers, then, foreseeing the effect of a larger participation on the expected price, would reduce rather than increase their optimal participation. Hence, in this case the expected price and the participation rate vary in opposite directions: a reduction in the perception bias would reduce the expected price and increase the level of participation.

Finally, as the perception bias varies in some cases we may move from one equilibrium to another. If the starting point is equilibrium 1.3 with full participation of high types, a reduction in k, making the low types less optimistic, at some point may move them to participate partially (1.2) or fully (1.1). In this case the free market expands as k falls. Since all the equilibria are along the same (upward sloping) curve $\widetilde{r}_1(\theta_1^*)$, the increase in participation is associated with a higher (less competitive) average price.

In the multiple equilibria case described in Proposition 2, following a reduction in k the equilibrium with partial participation of low types (2.3) always exists, with an increase in participation and expected prices. The equilibrium with partial participation of high type (2.1) converges, as k falls, to the one with full participation of high types (2.2), with a reduction in the expected price and an increase in participation. For lower values of k only equilibrium 2.3 survives. Hence, the fall in the perception bias may initially reduce the expected price, if equilibrium 2.1 is implemented (moving along $\widetilde{r}_2(\theta_1^*)$). We then have a discrete jump down in the expected price and a sharp increase in participation when only

[28] Our discussion therefore, applies also to a richer specification of the model where the perception bias of the two types of non shoppers move independently rather than being linked through k. Indeed, all that matters in the comparative statics with respect to the perception bias is how the perceived price p_0^i of the marginal type changes.

equilibrium 2.3 exists. As k further decreases, the equilibrium converges from below to either partial or full participation and an expected price $E^*(p) \leq p_R$. [29]

6.2 Changes in the search cost c

We move now to the analysis of equilibria when the search cost changes. A variation in the search cost does not affect the curve $\widetilde{\theta}_1(r)$ but shifts the locus $\widetilde{r}(\theta_1)$ moving to contours corresponding to a higher level of c. The following Lemma describes the effects on the participation rate and the expected price.

Lemma 7: *In the equilibria with* ***full participation*** *of both types or high types only, if the entry condition is not binding, the participation rate is not affected by the level of the search cost while the expected price is increasing in c in equilibrium* 1.1 *and* 1.3 *and decreasing in equilibrium* 2.2.

In the equilibria with ***partial participation*** (1.2, 1.4, 2.1, 2.3, 3.1, 3.3) *both the expected price $E^*(p)$ and participation rate θ_1^* are decreasing in the search cost c. Equilibria with full participation and a binding entry condition ($p_R - k = r(1)$ or $p_R + k = r(\frac{1}{2})$), behave as partial participation equilibria.*

The effects of an increase in the search costs are quite different if the equilibria involve full or partial participation. In this latter case less costly search makes non-shoppers more willing to search, increasing their participation. Shoppers then reduce their weight in the composition of active consumers making firms competing less aggressively. With full participation of high or both types, at the margin search costs do not affect the decision to search of active non-shoppers and all the effects take place through the impact on expected prices. Less costly search pushes up the expected price if the equilibrium is in the decreasing segment of $\widetilde{r}(\theta_1)$, with an opposite adjustment in the other case. The following table summarizes the results and connects them to the different equilibria.

Table 3 - Comparative statics with respect to c

$c \downarrow$	$r^*(\theta_1^*) = r_1^*(\theta_1^*)$	$r^*(\theta_1^*) = r_2^*(\theta_1^*)$
$\theta_1^* = 1, \theta_1^* = \frac{1}{2}$ full participation	$E^*(p) \downarrow$, θ_1^* constant eq. 1.1 and 1.3	$E^*(p) \uparrow$, θ_1^* constant eq. 2.2
$\theta_1^* \in \left(0, \frac{1}{2}\right), \theta_1^* \in \left(\frac{1}{2}, 1\right)$ partial participation	$E^*(p) \uparrow$, $\theta_1^* \uparrow$ eq. 1.2, 1.4, 2.3, 3.3	$E^*(p) \uparrow$, $\theta_1^* \uparrow$ eq. 2.1, 3.1

A fall in the search cost may also drive the market through different equilibria. A natural sequence may be starting from an equilibrium with full participation of high types (1.3), then moving to partial participation of low types (1.2)

[29] More precisely, with full participation the expected price is $E^*(p) = \widetilde{r}_1(1) - c$. When instead low types partially participate the expected price is $E^*(p) = p_R - c$. Since $\widetilde{r}_1(1) \leq p_R$ the expected price is not lower in the second case.

and ending with full participation of all non-shoppers (1.1). The participation initially remains constant with a fall in the expected price. Once moving to partial participation of low types non-shoppers' engagement increases together with the expected price. With full participation of low types (1.1) a further drop in search costs leads to a fall in the expected prices. An interesting dynamics takes place also in the case of multiple equilibria described in Proposition 2. When c falls the equilibrium with partial and full participation of high types (2.1 and 2.2) converge with an increase in the expected prices. At some low level of the search costs these equilibria disappear, and the only equilibrium entails low type partial participation (2.3), an initial drop in prices, a jump up in participation. As c further decrease the equilibrium converges to full participation. [30]

Hence, summing up the findings on comparative statics, different patterns of adjustment in prices and participation can take place. In most cases a reduction in the perception bias or in the search cost tend to increase participation. However the dynamics of expected prices may be non monotonic. When frictions of any kind become small, participation increases. If the perception bias shrinks the adjustment entails increasing prices whereas a reduction in search costs makes the prices fall.

7 Lifting the regulated price

The Italian Government, following a general tendency in the European Union, has planned to lift the regulated contract in the near future, in order to complete the liberalization process. The deadline was initially set at July 2018, but the evidence of a households market still entrenched on the regulated tariff and the poor performances realized on the free market by many switchers have recently prompted to move the deadline one year forward to July 2019.

Our model can be useful to predict the state of the market at the time the default contract is lifted, offering a benchmark to compare with the equilibrium in the market after the lift. This way we may identify different adjustments in the expected price and level of participation moving from one regime to the other.

There are two main changes in the model to analyze the environment after lifting the regulated contract. Once this latter is dropped, no consumer has an outside option $v - p_0^i$ associated with the regulated price. In the new environment the outside utility if no contract is signed on the free market corresponds to receiving no electricity service, and it is much lower, e.g. normalized to zero.

Secondly, in our benchmark model low and high type non-shoppers have different perceptions of the price of their legacy regulated contract. All households, including non-shoppers, instead, correctly perceive the proposed price when searching for new offers. We justified this behavioral trait with non-shoppers' mistakes in correctly remembering, or retrieving from the electricity

[30] If $c \rightarrow 0$ search cost do not matter anymore and the market equilibrium converges to the Bertrand outcome.

bill, the current price of a contract they subscribed in the past. These errors, instead, do not arise in our set up when non-shoppers face directly the price of a new commercial offer. Turning to an environment where the legacy regulated contract is dropped and only new offers matter, these perception biases play no role since there is no legacy contract in place, and the distinction between low and high type non-shoppers looses significance. The only remaining source of heterogeneity among consumers refers to search costs and the distinction between shoppers and non-shoppers. Then, a non-shopper will search if $v - E(p) - c \geq 0$. Since v is very high, it is reasonable to guess that all non-shoppers will participate, implying $r(1) \leq v$.

This set-up corresponds to Janssen et al. (2005) with full participation. Hence, we use our model to describe the market before the regulated contract is lifted and Janssen's to analyze the market equilibria in the first phase once the default contract is removed.[31] Since our model encompasses Janssen's the comparison of before and after lifting equilibria is feasible.

Let BL denote the relevant variables before liberalization (our model) and AL refer to those after liberalization (Janssen's model). Then the Φ functions have the form

$$\widetilde{\Phi}_{BL}(\theta_1^{BL}) = \begin{cases} 1 - \int_0^1 \frac{dy}{1+\frac{bn}{\theta_1^{BL}}\widetilde{y}_{BL}^{n-1}} & \text{for} \quad r_{BL} \leq p_R \\ 1 - \widetilde{y}_{BL} - \int_{\widetilde{y}_{BL}}^1 \frac{dy}{1+\frac{bn}{\theta_1^{BL}} y_{BL}^{n-1}} & \text{for} \quad r_{BL} > p_R \end{cases} \tag{12}$$

and

$$\widetilde{\Phi}_{AL}(\theta_1^{AL}) = 1 - \int_0^1 \frac{dy}{1 + \frac{bn}{\theta_1^{AL}}\widetilde{y}_{AL}^{n-1}} \tag{13}$$

respectively. Likewise, $\widetilde{y}_{BL} = \widetilde{y}(p, r_{BL}, \theta_1^{BL})$ and $\widetilde{y}_{AL} = \widetilde{y}(p, r_{AL}, \theta_1^{AL})$. We can notice that $\widetilde{\Phi}_{AL}(\theta_1) = \widetilde{\Phi}_{BL}(\theta_1)$ for $r_{BL} \leq p_R$, where the two models are equivalent. Figure 5 shows the reservation price and participation curves in the BL and AL environments. For $r_{BL} \leq p_R$ the reservation price in the two cases is the same ($r_{BL}(\theta_1) = \widetilde{r}_1(\theta_1) = r_{AL}(\theta_1)$) whereas for $r_{BL} > p_R$ the curve $r_{AL}(\theta_1)$ is increasing and below $r_{BL}(\theta_1)$. The participation curve after-lifting is flat at $r = v$ for any $\theta_1 \in [0, 1]$ and then vertical.

Figure 5 draws a comparison between the before- and after-lifting environments taking as an example the multiple equilibria in Proposition 2, that allow to discuss the cases when before-lifting the market equilibrium entails partial participation (2.1) or full participation (2.2) of high type or partial participation

[31] Janssen's model applies to the initial phase after lifting the regulated price, in which all consumers have to choose a new contract and they share the same outside option. The mixed strategies generate price dispersion. Then, in the choice of searching or sticking to the current contract consumers in a second period are characterized by different outside options. This is the key feature of our benchmark model, that may be the starting point to analyze the price dynamics in the free market. This topic is left to future research.

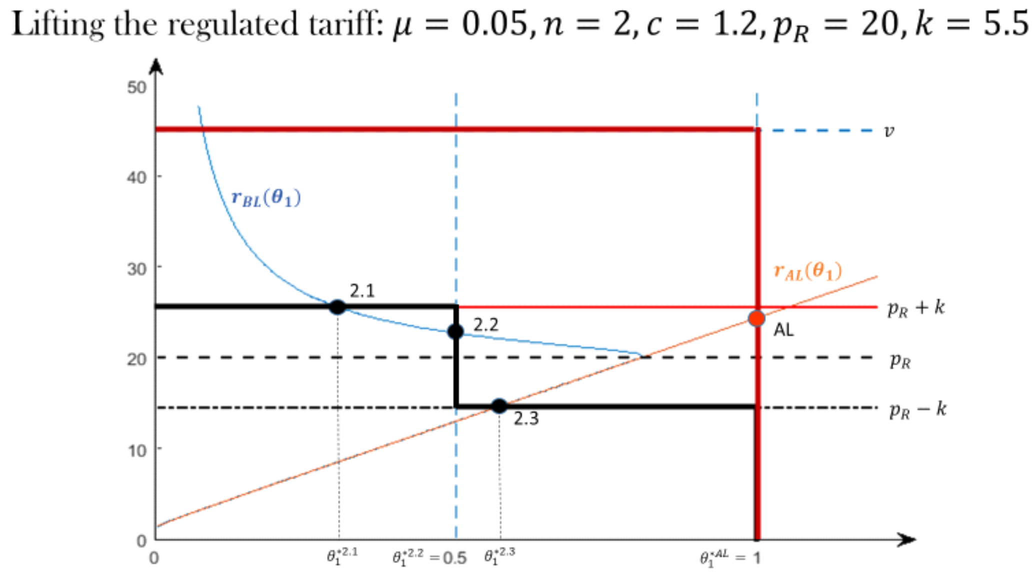

Figure 5: Lifting the regulated tariff

of low types (2.3). Starting with this latter case, the BL and AL equilibria lie on the same curve and we can predict that the expected price, following the increase in participation after lifting, will increase. As discussed in Janssen et al. (2005) (see fig. 5(b) in their paper), a large number of non-shoppers prompts firms to raise prices to capture rents from them. If instead the BL equilibrium entails partial or full participation of high types, it is evident from the figure that the expected price, depending on the relative position of the curves, may increase ($r_{BL}(\theta_1^{2.1})$ or $r_{BL}(\theta_1^{2.2})$ larger than $r_{AL}(1)$) or decrease. A comparison of the before- and after-lifting expected price can be easily extended to the other equilibrium configurations, taking into account of the relationships between $r_{BL}(\theta_1)$ and $r_{AL}(\theta_1)$ and the different participation patterns of non-shoppers in the two market environments.

Hence, dropping the default contract involves an increase in participation and an increase in the expected price if, in the BL equilibrium the participation is large (low types partially participate). If instead only high type participate before lifting, the increase in participation can go along with an increase or a decrease in the expected price.

To assess further developments in the market after full liberalization, it may be useful to consider possible changes in the parameters that may be affected by market opening. For instance, firms may enter driven by the expansion of the market after the regulated price is lifted, or consumers' learning in the new free market environment may reduce the level of search costs and increase the percentage of shoppers. In a market with full participation, a result in the search sequential literature is that an increase in the number of firms under full participation leads to higher prices: since the probability of being selected by shoppers falls in the number of firms, these latter will be more focused, in selecting their mixed strategies, on non-shoppers (Stahl '89, Janssen et al. '05). Conversely, a reduction in search costs and an increase in the fraction of shoppers reduce the average price of the equilibrium mixed strategies (Stahl '89, Janssen et al. '05). After all, these beneficial effects of consumers' activism are the ultimate motivation to completely liberalize the demand side of the retail markets.

8 Conclusions

The Italian retail electricity market after a decade is still characterized by low participation of households, that remain to a large extent with the default regulated contract, a low switching rate, price dispersion, an average price for customer in the free market more costly than the regulated one. In this paper we have provided additional evidence analyzing retailers' strategies and offers to new clients that move in the free market, quantifying the gains from switching from the regulated contract to the free market. Although picking up a contract at random would expose to a moderate loss compared with carrying on with the regulated contract, potential gains from an intelligent choice are significant in absolute and percentage terms. The low participation rate in the market suggests that households do not fully exploit these bargains. We have also highlighted that in the free market retailers' strategies entail offers that are more costly than the regulated one. Unexploited bargains and potential rip-offs require therefore an explanation.

While price dispersion and low participation can be obtained in a standard model of consumer search, firms' commercial strategies offering contracts more costly than the default one are inconsistent with this setting. Our explanation looks at perception biases of households on the current regulated price they are paying. We argue that alternative ingredients of transaction cost or behavioral bias approaches would not account for this evidence.

To this end we have built up a model of the electricity market where consumers search for price quotes and firms compete in prices, generalizing Janssen et al. (2005). Consumers are assumed to be heterogeneous under two dimensions, referred to their search costs and the perception of the current regulated price they pay. Shoppers have zero search costs and a correct perception of the regulated price of the running contract. Non-shoppers, instead, have positive search costs and a bias in the perception of the current regulated price. High (upward bias) and low (downward bias) non shoppers and shoppers form the demand side. In equilibrium firms randomize generating price dispersion. According to the parameters, including the number of firms, the share of shoppers, the size of the cognitive bias and the search cost, the equilibria differ in terms of participation of non-shoppers, the maximum and minimum price of the distribution and the average price paid. We may have mixed strategies over a continuous support and others with a continuous support and an atom at a high price, that is chosen only by high type non-shoppers, an innovative result in the search literature. Price dispersion is consistent with the wide variety of prices offered in the free market. Some equilibria, where only high type non-shoppers and shoppers participate, are characterized by prices below the regulated one (bargains from switching) but also a high price well above it (rip-offs from switching), as the evidence found looking at retailers' strategies in the free market.

Our results have some interesting policy implications. The policy debate on improving the functioning of retail energy markets tends to focus on reducing search costs on *new* prices. Our findings show that a correct perception of the *current* price is as important in shaping the market equilibrium. Consumers' information, both in terms of reducing the search cost on new offers and the perception bias on the current price, are the key elements the policy should address. However, the effects of a reduction in the search cost and perception bias may have different effects on the level of participation and the expected price, depending on the initial equilibrium configuration. If search costs and perception bias are significantly reduced we may expect the market equilibrium to move towards an increase in the participation of (low type) non-shopper and in the expected price.

Our results are useful also to evaluate the possible effects of lifting the regulated price, a move to complete liberalization that is under discussion both in Italy and in the European Union. In the new environment participation increases. The effect on expected prices instead depends on the initial state of the market before lifting the regulation. If participation is already large we expect an increase in prices, while different outcomes may arise if the market is small at the time the regulation is lifted.

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Appendix I: Methodology to find the best and worst offers

Our analysis resembles the procedure carried out by the UK Competition and Markets Authority in its Energy Market Investigation (2016). Our procedure can be summarized in three steps:

1. Define reference consumer profiles.
2. Compute the annual retail electricity bills for the reference consumer profiles and different levels of annual consumption and peak/off peak allocation and identify the lowest and the most expensive offers in the free market.
3. Calculate the difference between the standard regime tariffs and those identified at point 2 to find potential gains and losses.

Consumer profiles are defined by the power installed, the level of annual consumption, its allocation between peak and off peak hours and preferences for certain contractual features.

Power installed. Regarding the power installed, the Regulator identifies three main groups of domestic consumers:[32] (1) Domestic residents with installed power below or equal 3 KW: this group accounts for $76,9\%$ of the total volumes supplied to domestic consumers under the standard regime tariff. About 15 million households are supplied with this power system[33]; (2) Domestic residents above 3 KW: these consumers represent a small share (around

[32] After a reform of the structure of the regulated parts of the energy bill, the Regulator has introduced a new classification, that distinguishes only between Resident and Non-Resident domestic consumers. Since we are using consumption data from 2015, when the previous classification was in place, we use the former.

[33] For this category, we collect data for consumers with 1.5 kW and 3 kW of installed power

5%) of the total pool of domestic consumers subscribed to the standard regime tariff, but tend to have higher consumption volumes than those under 3 KW. Indeed, their annual mean consumption is 3915 KWh/year against the 2000 KWh/year of the first group of consumers, and the volumes involved represent around 11.5% of the total volumes supplied through regulated contract; (3) Non-resident domestic consumers: usually owners of second homes, this group counts nearly 5 million households. Mean consumption is considerably lower than for the previous two categories, being around 957 KWh/year. Among these three groups we focus on the first one, that amounts for a substantial part of households served according to the regulated contract.

Annual Consumption. For each of these categories, the Regulator divides consumers into seven classes, based on their consumption volumes. We adopt this level of disaggregation and for each class we impute the corresponding average annual consumption to calculate retail bills and the level of potential gains and losses for each class of customers.

Peak/off Peak allocation. The widely offered two-prices tariff shows two different prices for two time slots of the day. Slot F1 consists of hours between 8 and 19 from Monday to Friday, while slots F2-F3 consists of the hours between 19 and 8 (F2), and the weekends (F3). We defined a second set of scenarios, assuming a different time pattern in consumption between day and night. We consider the following three cases. In case **(A)** *85%* of household annual consumption occurs in the evening hours and during weekends (F2-F3 slot). This percentage falls down to 70% in case **(B)** and to 50% in case **(C).**

Contract scenarios. Each scenario restricts the set of contracts to which the consumer may be willing to switch according to specific restrictions that affect the complexity of the new tariff, risk aversion, attachment to a specific payment method. These scenarios are defined as follows: **(1)** *Unconstrained*: the most flexible scenario, it includes all the types of contracts offered in the market. **(2)** In this scenario, it is assumed that consumers prefer a *tariff with a single rate*, that does not change depending on the hour of the day when energy is consumed . These tariffs may be easier to interpret and they may be preferable for those consumers that are uncertain about their allocation of daily consumption. **(3)** In this scenario, the energy price is fixed for 12 months. **(4)** This scenario restricts consumer's choice to contracts that allow *payment through postal paying-in slips*. In Italy, this is still the most common type of payment, and it is allowed for the standard regime tariff but not for many other offers available on the market, who restrict payment method to direct debit or credit card.

To carry out comparison, we exploited the algorithm of the Italian PCWs run by the Authority:TrovaOfferte.[34] On these website, consumers can indicate their zip codes, their current consumption and their preferences on tariff characteristics. Then, the algorithm of the PCW calculates the annual bill to be paid under the standard regime tariff and the annual bill to be paid for each of

[34] available at http://trovaofferte.autorita.energia.it/trovaofferte

the tariffs available on the free market [35]. After the calculation, we adjusted the prices of the offers by including relevant discounts that were not previously included by the PCW[36] and, for Scenario 4, the costs of paying by postal slip (when there is the possibility to do so). As far as concerns payment costs, we integrated information on the TrovaOfferte with the one available on other PCWs and on retailers' websites.

We calculated annual bills for each consumption group (as identified by the NRA in terms of consumption volumes) belonging to the "Domestic resident below 3K". For each consumption group we calculated bills by allocation of total consumption between peak and off peak and by contractual preferences. For *regulated contract* bills, we kept and used the data concerning two-rate time-of-the-day tariffs, that covers more than 95% of domestic households (AEEGSI, 2016).[37] Finally, we selected tariffs that would be available for an average consumer searching online from the city of Milan: this means we ruled out tariffs reserved to restricted groups of consumers (such as entrepreneurs already subscribed to a given company for business purposes) and to consumers with single-rate meters. On the other hand, we kept in our sample tariffs that could only be subscribed through online channels. Finally, we excluded tariff indexed to the wholesale market that used the wholesale price of a single month (January or February 2017) to calculate the annual bill, because they overestimated the annual energy bill.

Once the regulated price and the ones of the most and the least convenient tariffs on the free market are identified, the calculation of the potential gains and losses from switching consists in a simple difference between the two measures. The strategy used simulates the result obtained by a domestic consumer subscribed to the regulated tariff that would search online for an alternative tariff. The annual regulated bill and free market bills are calculated, through the PCW, for each consumption category in each scenario defined above.

[35] We also carried out searches on a privately run PCW, Sos Tariffe. Since findings on the two websites are not always consistent, both in terms of prices reported and tariff classification, we have only used data from TrovaOfferte.

[36] We included:

- una tantum bonuses that could be reaped by a consumer subscribing to a given offer for 12 months;
- discounts related to payment by direct debit;
- other discounts available to all consumers subscribing a given offer, such as a fixed discount on the listed energy price.

For offers lasting 24 months, we used a pro-quota-die criterion. For example, for a two.year offer providing a fixed 50 euros discount , we subtracted 25 euros to the estimated annual bill.

[37] Also, dual fuel tariffs are excluded for the analysis. If they were included, also gas tariffs would have to be analyzed, and this goes beyond the scopes of this research. Moreover, a study on national scale would be much more complicated: gas contracts are not homogeneous on the Italian territory, and their availability depends on the zip code.

Appendix II: Proofs

Proof of Lemma 1. Consider a non-shopper who has searched $t \geq 0$ times. She has a set of $t+1$ available prices, the t quotes and the initial signal p_0^i, with $\underline{p}_t^i$ being the lowest one, $i = L, H$. Her utility, gross of the sunk search costs $t * c$, from purchasing after t searches is $U_t^{ij} = v - \underline{p}_t^i$. Furthermore, given the symmetric mixed strategies of the firms, $F(p)$, the expected utility from searching a further offer p incurring an additional search cost is

$$U_{t+1}^{ij} = v - \int_{\underline{p}}^{\underline{p}_t^i} p\hat{f}(p)dp - \sum_{pl} p_l \Pr(p = p_l | p \leq \underline{p}_t^i) - \underline{p}_t^i \left(1 - F(\underline{p}_t^i)\right) - c,$$

where we are taking into account that any quote above $\underline{p}_t^i$ obtained with a further search would be rejected in favor of $\underline{p}_t^i$. Then, the condition $U_{t+1}^{ij} \geq U_t^{ij}$ can be rewritten as:

$$\int_{\underline{p}}^{\underline{p}_t^i} (\underline{p}_t^i - p)\hat{f}(p)dp + \sum_{p_l} (\underline{p}_t^i - p_l) \Pr(p = p_l | p \leq \underline{p}_t^i) - c \geq 0. \tag{14}$$

that is the expected savings from one more search, computed according to the strategies of the firms, to be not lower than the search cost. Both the integral and the summation in (14) are strictly increasing in $\underline{p}_t^i$ for a given price distribution. Then, the consumer is indifferent between searching one more time or purchasing the lowest price among those available if $\underline{p}_t^i = r$, where r is defined as in the statement. ■

Proof of Lemma 2. Since non-shoppers differ in terms of the signal received, with the low types believing to currently pay $p_0^L = p_R - k$ and the high types over-estimating their regulated electricity bill $p_0^H = p_R + k$, their behavior may differ. Non-shoppers of low or high type decide to run a first search if condition (2) holds. Since the integral in (2) is increasing in p_0^i, high types expect a higher gain from entering the market. Hence, $\theta_1^H < \theta_1^L$ cannot occur in equilibrium. ■

Proof of Lemma 3. The reservation price of non-shoppers r depends, as already argued, on the pricing strategy $f(p)$ which, in turn, is affected by the participation and search behavior of non-shoppers. Then, no firm would post a price above r since it would be rejected, inducing a further search. Then $\bar{p} = r$. This implies that if a non-shopper decides to search once, then the sampled price will be accepted. The share of active non-shoppers is therefore given by θ_1. ■

Proof of Lemma 4. Suppose that there exists a PBE where only shoppers participate ($\theta_1 = 0$). Then, since shoppers choose the lowest price firms compete a la Bertrand and they all post with probability 1 a price equal to zero. But then (2) for high types boils down to $p_R + k - c > 0$ by Assumption 1, and at least the high types find it convenient to search, and $\theta_1 > 0$. ■

Proof of Lemma 5. If the price distribution $f(p)$ had an atom at some $p \in [\underline{p}, p_R]$ then another firm could discretely increase its profits by undercutting the atom. Hence, the optimal price distribution $\widetilde{f}(p)$ must be atomless in the interval where shoppers could accept a posted price after sampling all prices.

Consider first the case when, for given θ_1, $p_R \geq r$. The reservation price depends on θ_1 through $F(.)$, and we omit the reference to streamline the notation. Since no firm sets a price higher than r, then the expected profits when the other firms follow the symmetric mixed strategy $F(p)$ are

$$\pi_h(p, F(p)) = p\left[\frac{(1-\mu)\theta_1}{n} + \mu(1-F(p))^{n-1}\right] \quad \text{if} \;\; p \in [0, \overline{p}]. \tag{15}$$

Setting

$$\pi(p, F(p)) = p\left[\frac{(1-\mu)\theta_1}{n} + \mu(1-F(p))^{n-1}\right] = \pi(\overline{p}, F(p)) = \overline{p}\frac{(1-\mu)\theta_1}{n}$$

and solving we get the optimal mixed strategy for a given participation rate of non-shoppers θ_1:

$$\widetilde{F}(p; r, \theta_1) = 1 - \left[\frac{\theta_1(r-p)}{nbp}\right]^{\frac{1}{n-1}} \quad \text{for} \quad p \in [\underline{p}, r]$$

as in the statement. Finally, setting $\widetilde{F}(p) = 0$ and solving for p we get the expression of $\underline{p}$ that is clearly lower than p_R when $r \leq p_R$.

Next consider the case $p_R < r$. For prices $p \in (p_R, r]$ shoppers, after sampling the n prices, remain with the regulated contract whereas active non-shoppers who have sampled p accept this quote. Then the optimal price distribution $\widetilde{f}(p)$ is continuous on $p \in [\underline{p}, p_R]$ and has an atom at r that is chosen with probability $1 - \widetilde{F}(p_R)$. The optimal mixed strategy for $p \in [\underline{p}, p_R]$ is then

$$\widetilde{F}(p; r, \theta_1) = 1 - \left[\frac{\theta_1(r-p)}{nbp}\right]^{\frac{1}{n-1}} \quad \text{for} \quad p \in [\underline{p}, p_R].$$

with $\underline{p} = \frac{(1-\mu)\theta_1 r}{\mu n + (1-\mu)\theta_1}$. Then, $\mu > \overline{\mu}(\theta_1)$ implies $\underline{p} < p_R$.

Finally, given that the only atom of the optimal mixed strategy is at $p = r$, we can rewrite the definition of the reservation price (1) as

$$\int_{\underline{p}}^{p_R} (r-p)\widehat{f}(p; r, \theta_1)dp - c = 0.$$

where $\widehat{f}(p; r, \theta_1)$ is the continuous component of the optimal mixed strategy. Rearranging, this yields (6) where, according to Lemma 2, we use the fact that

$\bar{p} = r$. (6) is straightforward when $r \leq p_R$. When instead $r > p_R$ the expected price is

$$\widetilde{E}(p; r, \theta_1) = \int_{\underline{p}}^{p_R} p\widehat{f}(p; r, \theta_1)dp + \left(1 - \widetilde{F}(p_R; r, \theta_1)\right) r$$

The reservation price, in turn, is implicitly defined by the condition $\int_{\underline{p}}^{p_R}(r - p)\widehat{f}(p)dp - c = 0$, or

$$\widetilde{F}(p_R)r = \int_{\underline{p}}^{p_R} p\widehat{f}(p)dp + c.$$

Then, adding and subtracting $\left(1 - \widetilde{F}(p_R\right) r$ to the right hand side and rearranging we get $r = \widetilde{E}(p; r, \theta_1) + c$. ■

Proof of Proposition 1. Suppose there exists a PBE in which, given θ_1, the firms adopt a mixed strategy $\widetilde{F}(p, \theta_1)$ such that $r = \widetilde{r}_2(\theta_1)$, that is the reservation price is defined in the decreasing portion of $\widetilde{r}(\theta_1)$. If $\widetilde{r}_2(\frac{1}{2}) > p_R + k > p_R - k$ both H and L would carry on with their (perceived) regulated price and would not participate. Then, according to Lemma 4, a PBE does not exist. Then, if an equilibrium exists the mixed strategy must be associated with $r = \widetilde{r}_1(\theta_1)$, that is, with an equilibrium mixed strategy such that the reservation price is on the increasing portion of $\widetilde{r}(\theta_1)$. Since $\widetilde{r}_1(\theta_1)$ is increasing in θ_1 and in equilibrium $p_0^i \geq \widetilde{r}_1(\theta_1)$ if type $i = H, L$ participate, then there is a single θ_1 that solves the inequality, implying uniqueness. Let us now consider the different cases.

(1.1) If $\widetilde{r}_1(1) < p_R - k$ both types search, and therefore $\theta_1^{1.1} = 1$. If $\widetilde{r}_1(1) = p_R - k$ the low types are indifferent and randomize. If $\theta_1^L < \frac{1}{2}$ then $\theta_1 < 1$ and $\widetilde{r}_1(\theta_2) < p_R - k$ implying that the L types strictly prefer to search. Hence, even in this case $\theta_1^L = \frac{1}{2}$ and $\theta_1 = 1$. Then, a unique symmetric PBE exists with $\theta_1^{1.1} = 1$ and $\widetilde{F}(p, \theta_1^{1.1})$ as defined in (5. Since $\widetilde{r}_1(1) < p_R$ the mixed strategy is defined on the support with no atoms with $\overline{p} = \widetilde{r}_1(1)$.

(1.2) If $\widetilde{r}_1(\frac{1}{2}) \leq p_R - k < \widetilde{r}_1(1) < p_R + k$ then H types fully participate and, since $\widetilde{r}_1(\theta_1)$ is increasing, there exists a unique $\theta_1^{1.2} \in (\frac{1}{2}, 1)$ such that $p_R - k = \widetilde{r}_1(\theta_1^{1.2})$ with L types participating with probability $\theta_1^{1.2} - \frac{1}{2}$. If $\widetilde{r}_1(\frac{1}{2}) = p_R - k$ the low types are indifferent and randomize. If $\theta_1^L > 0$ then $\theta_1 > \frac{1}{2}$ and $\widetilde{r}_1(\theta_1) > p_R - k$ implying that the low types prefer to stick on the regulated contract. Hence in this case $\theta_1^L = 0$ and $\theta_1^{1.2} = \frac{1}{2}$. Since $\widetilde{r}_1(\theta_1^{1.2}) < p_R$ the mixed strategy is defined on the support with no atoms with $\overline{p} = \widetilde{r}_1(\theta_1^{1.2})$.

(1.3) If $p_R - k < \widetilde{r}_1(\frac{1}{2}) < p_R + k$ the H types fully participate and the L type stay out, then $\theta_1^{1.3} = \frac{1}{2}$. If $\widetilde{r}_1(\frac{1}{2})$ is lower (larger) than p_R the mixed strategy is defined on the support with no (one) atom with (at) $\overline{p} = \widetilde{r}_1(\theta_1^{1.2})$. If $\widetilde{r}_1(\theta_1^{1.2}) > p_R$, i.e. the mixed strategy has an atom, the condition on μ ensures that the continuous support is non empty.

(1.4) Finally, if $p_R + k \in \left(c, \widetilde{r}_1(\frac{1}{2})\right]$ there exists a unique $\theta_1^{1.4}$ such that $\widetilde{r}_1(\theta_1^{1.4}) = p_R + k$, corresponding to the probability that the H type enters.

Since $\widetilde{r}_1(\theta_1^{1.4}) > p_R$ the mixed strategy has an atom at $\widetilde{r}_1(\theta_1^{1.4})$. The condition on μ ensures that the continuous support is non empty ■

Proof of Proposition 2. If $\theta_1^D \geq \frac{1}{2}$ the reservation price $\widetilde{r}(\theta_1)$ and its higher value $\widetilde{r}_2(\theta_1)$ are defined for $\theta_1 = \frac{1}{2}$. Then suppose $\widetilde{r}_2(\frac{1}{2}) < p_R + k$. The first equilibrium lies at the point of intersection of $p_R + k$ and $\widetilde{r}_2(\theta_1)$. Since $\widetilde{r}_2(\theta_1)$ is decreasing in θ_1 and $\widetilde{r}_2(\frac{1}{2}) < p_R + k$ there exists a $\theta_1^{2.1}$ such that $\widetilde{r}_2(\theta_1^{2.1}) = p_R + k$, the low types remain with the regulated contract while the high types randomize. The equilibrium mixed strategy has an atom at $\widetilde{r}_2(\theta_1^{2.1}) > p_R$ and the condition on μ ensures that the continuous support is non empty. In the second equilibrium all the high types participate since $p_R - k < \widetilde{r}_2(\frac{1}{2}) < p_R + k$ whereas the low types stay out. Since $\widetilde{r}_2(\theta_1) > p_R$ the equilibrium mixed strategy has an atom at $\widetilde{r}_2(\frac{1}{2})$ and the condition on μ ensures that the continuous support is non empty . In the third equilibrium since $\widetilde{r}_1(\frac{1}{2}) < p_R - k$ and $\widetilde{r}_1(\theta_1)$ is increasing there exists a $\theta_1^{2.3} \in \left(\frac{1}{2}, 1\right]$ such that $\widetilde{r}_1(\theta_1^{2.3}) \leq p_R - k$, the high types fully participate and the low types randomize, fully participating ($\theta_1^{2.3} = 1$) if $\widetilde{r}_1(\theta_1^{2.3}) < p_R - k$. ■

Proof of Proposition 3. If $\theta_1^D < \frac{1}{2}$ the reservation price is defined only for $\theta_1 \leq \theta_1^D$ and therefore in equilibrium only H-types will partially participate. If $\widetilde{r}(\theta_1^D) < p_R + k$, since $\widetilde{r}(\theta_1)$ is decreasing above $\widetilde{r}(\theta_1^D)$ there exists a $\theta_1^{3.1} < \theta_1^D$ such that $\widetilde{r}_2(\theta_1^{3.1}) = p_R + k$. The high types are indifferent and randomize choosing $\theta_1^H = \theta_1^{3.1}$. Since $\widetilde{r}_2(\theta_1^{3.1}) > p_R$ the equilibrium mixed strategy has an atom at $\widetilde{r}_2(\theta_1^{3.1})$. If $\widetilde{r}(\theta_1^D) = p_R + k$ the high types randomize entry with probability θ_1^D and the mixed strategy has an atom at $\widetilde{r}(\theta_1^D)$. Finally, if $\widetilde{r}(\theta_1^D) > p_R + k$ the equilibrium partial participation of high types is $\theta_1^{3.3}$ such that $\widetilde{r}_1(\theta_1^{3.3}) = p_R + k$ and the mixed strategy has an atom at $\widetilde{r}_1(\theta_1^{3.3})$. In all three cases the condition on μ ensures that the continuous support is non empty. ■

Proof of Lemma 6. Since in equilibrium

$$E^*(p) + c = r^*(\theta_1^*) \leq p_0^{i*}, \tag{16}$$

if $r^*(\theta_1^*) < p_0^{i*}$, as it is in the equilibria 1.1, 1.3 and 2.2 where type i^* non-shoppers fully participate, (**??**) holds as an inequality and a marginal variation in p_0^{i*} due to a change in k does not affect $E^*(p)$ nor θ_1^*. When instead $r^*(\theta_1^*) = p_0^{i*}$ we have $E^*(p) + c = p_0^{i*}$ and therefore

$$sign\frac{\partial E^*(p)}{\partial k} = sign\frac{\partial p_0^{i*}}{\partial k} \begin{cases} > 0 & \text{if } i^* = H \\ < 0 & \text{if } i^* = L \end{cases}.$$

The first case occurs in equilibria 1.4, 2.1, 3.1, 3.2 and 3.3, where the marginal consumer is the high type and an increase in the bias k moves up the equilibrium expected price. An increase in k instead reduces the equilibrium expected price in the equilibria 1.2 and 2.3 where the marginal consumer is the low type.

Turning to the equilibrium *participation rate*, when $r^*(\theta_1^*) = p_0^{i*}$ we have $\theta_1^* = r^{*-1}(p_0^{i*})$ and therefore

$$\frac{\partial\theta_1^*}{\partial k} = \frac{\partial\theta_1^*}{\partial r}\frac{\partial p_0^{i*}}{\partial k}.$$

Then:

$$sign\frac{\partial\theta_1^*}{\partial k} = sign\frac{\partial\theta_1^*}{\partial r} * sign\frac{\partial p_0^{i*}}{\partial k}.$$

In this case two effects affect the impact of a variation in the bias k on the equilibrium participation. Whether the marginal type is low or high ($sign\frac{\partial p_0^{i*}}{\partial k}$), as discussed above, and whether the equilibrium occurs in the increasing or decreasing part of the locus $r(\theta_1)$. In the former case $r^*(\theta_1^*) = r_1(\theta_1^*)$ and $sign\frac{\partial\theta_1^*}{\partial r} > 0$, as it is in the equilibria 1.2, 1.4, 2.3 and 3.3. When instead $r^*(\theta_1^*) = r_2(\theta_1^*)$ we have $sign\frac{\partial\theta_1^*}{\partial r} < 0$, a feature of equilibria 2.1 and 3.1. ■

Proof of Lemma 7. From consumers' optimal entry and firms optimal prices we know that, in equilibrium

$$E^*(p) + c = r^*(\theta_1^*) \leq p_0^i$$

If $r^*(\theta_1^*) = p_0^i$, we have partial participation of type i. Then

$$\frac{\partial r^*(\theta_1)}{\partial c} = \frac{\partial p_0^i}{\partial c} = 0$$

Hence

$$\frac{\partial(E^*(p) + c)}{\partial c} = \frac{\partial E^*(p)}{\partial c} + 1 = \frac{\partial r^*(\theta_1)}{\partial c} = 0$$

and

$$\frac{\partial E^*(p)}{\partial c} = -1 < 0.$$

Therefore, in equilibria with partial participation the equilibrium reservation price remains constant while average prices decrease following a marginal increase in c.

If instead $r^*(\theta_1^*) < p_0^i$, implying full participation of type i,

$$\frac{\partial E^*(p)}{\partial c} = \frac{\partial r^*(\theta_1^*)}{\partial c} - 1$$

Therefore, the effect of a marginal increase in the level of search costs depends on $\frac{\partial r^*(\theta_1^*)}{\partial c}$, namely, if $\frac{\partial r^*(\theta_1^*)}{\partial c} > 1$, then $c \uparrow \Rightarrow E^*(p) \uparrow$, whereas if $\frac{\partial r^*(\theta_1^*)}{\partial c} < 1$, then $c \uparrow \Rightarrow E^*(p) \downarrow$.

To study the sign of $\frac{\partial r^*(\theta_1^*)}{\partial c}$, we can start from condition (9), that is

$$G = r\widetilde{\Phi} = c$$

Differentiating

$$\frac{\partial r^*(\theta_1^*)}{\partial c}\widetilde{\Phi} + r\frac{\partial \widetilde{\Phi}}{\partial c} = 1$$

Since $\widetilde{\Phi}$ is a function of r, we can rewrite the above as

$$\frac{\partial r^*(\theta_1^*)}{\partial c}\widetilde{\Phi} + r^*\frac{\partial \widetilde{\Phi}}{\partial r}\frac{\partial r^*(\theta_1^*)}{\partial c} = 1 \Rightarrow \frac{\partial r^*(\theta_1^*)}{\partial c}(\widetilde{\Phi} + r^*\frac{\partial \widetilde{\Phi}}{\partial r}) = 1$$

Then

$$\frac{\partial r^*(\theta_1^*)}{\partial c} = \frac{1}{\widetilde{\Phi} + r^*\frac{\partial \widetilde{\Phi}}{\partial r}}.$$

When $r^* \leq p_R$, clearly $\frac{\partial \widetilde{\Phi}}{\partial r} = 0$ and, since $\widetilde{\Phi} < 1$, $\frac{\partial r^*(\theta_1^*)}{\partial c} > 1$. Hence, when $c \uparrow \Rightarrow E^*(p) \uparrow$.

When instead $r^* > p_R$ and the optimal price distribution is characterized by an atom, the effect of a change in the search cost on the optimal reservation price is less straightforward. In this case it is useful to note that $\widetilde{\Phi} + r\frac{\partial \widetilde{\Phi}}{\partial r} = \frac{\partial G}{\partial r}$ and that $\frac{\partial G}{\partial r} \in (0, \widetilde{\Phi})$ for $r^* < \widetilde{r}(\theta_1^D)$ and $\frac{\partial G}{\partial r} < 0$ for $r^* > \widetilde{r}(\theta_1^D)$. Hence,

$$\frac{\partial r^*(\theta_1^*)}{\partial c} = \frac{1}{\widetilde{\Phi} + r^*\frac{\partial \widetilde{\Phi}}{\partial r}} > 1 \quad \text{for} \quad r^*(\theta_1^*) < \widetilde{r}(\theta_1^D) \implies c \uparrow \Rightarrow E^*(p) \uparrow$$

while

$$\frac{\partial r^*(\theta_1)}{\partial c} = \frac{1}{\widetilde{\Phi} + r^*\frac{\partial \widetilde{\Phi}}{\partial r}} < 0 \quad \text{for} \quad r^*(\theta_1^*) > \widetilde{r}(\theta_1^D) \implies c \uparrow \Rightarrow E^*(p) \downarrow .$$

Moving to the optimal participation rate, when $r^*(\theta_1^*) = p_0^i$, considering that we can rewrite this as $r^*(\theta_1^*(c), c) = p_0^i$

$$\frac{\partial r^*(\theta_1^*)}{\partial \theta_1}\frac{\partial \theta_1^*}{\partial c} + \frac{\partial r^*(\theta_1^*)}{\partial c} = \frac{\partial p_0^i}{\partial c} = 0 \implies \frac{\partial \theta_1^*}{\partial c} = -\frac{\partial r^*(\theta_1^*)}{\partial c}\frac{\partial \theta_1^*}{\partial r}.$$

When $r^*(\theta_1^*) = r_1^*(\theta_1^*)$, $\frac{\partial r^*(\theta_1^*)}{\partial c} > 0$ and $\frac{\partial \theta_1^*}{\partial r} > 0$, hence $\frac{\partial \theta_1^*}{\partial c} = - * + * + < 0$. When instead $r^*(\theta_1^*) = r_2^*(\theta_1^*)$, $\frac{\partial r^*(\theta_1^*)}{\partial c} < 0$ and $\frac{\partial \theta_1^*}{\partial r} < 0$, hence $\frac{\partial \theta_1^*}{\partial c} = - * - * - < 0$. Then, in equilibria of partial participation, following a marginal increase in c participation always decreases. ■

Appendix III: The shape of $\widetilde{r}(\theta_1)$

Equation (9) can be thought as defining implicitly contours at c of the function $G(r, \theta_1) = r * \widetilde{\Phi}(r, \theta_1)$ in the (r, θ_1) space, and $\widetilde{r}(\theta_1)$ is the correspondence describing those contours, with

$$\left.\frac{dr}{d\theta_1}\right|_{G(r,\theta_1)=c} = -\frac{\partial G/\partial \theta_1}{\partial G/\partial r}.$$

The locus $\widetilde{r}(\theta_1)$ is therefore the optimal reservation price when the cost of search is c and the participation rate is θ_1, given the mixed strategy $\widetilde{F}(p;r,\theta_1)$. We show now that $\widetilde{r}(\theta_1)$ is initially increasing and then decreasing. Consequently, for each level of search cost $c \in (0, p_R+k)$ there are two values of $r(\theta_1)$ associated to each participation rate $\theta_1 \in (0, min[\theta_1^D, 1])$, $\widetilde{r}_1(\theta_1)$ and $\widetilde{r}_2(\theta_1)$, with $r_1 < r_2$, where θ_1^D is the participation rate at which $\widetilde{r}(\theta_1)$ changes slope.

For $r \leq p_R$ we have

$$G(r,\theta_1) = r * \widetilde{\Phi}(\theta_1) = r * [1 - \int_0^1 \frac{(1-\mu)\theta_1}{(1-\mu)\theta_1 + \mu n \widetilde{y}^{n-1}} dy]$$

where the term in square brackets is $\Phi(\theta_1)$, it does not depend on r and it is decreasing in θ_1. This case corresponds to Janssen et al. (2005) with partial participation, and

$$\left.\frac{dr}{d\theta_1}\right|_{G(r,\theta_1)=c} = -\frac{\partial G/\partial\theta_1}{\widetilde{\Phi}(\theta_1)} > 0.$$

When $r > p_R$ the function $G(r,\theta_1)$ can be written as

$$G(r,\theta_1) = r * [\widetilde{F}(p_R) - \int_{1-\widetilde{F}(p_R)}^1 \frac{(1-\mu)\theta_1}{(1-\mu)\theta_1 + \mu n \widetilde{y}^{n-1}} dy]$$

where the term in square brackets corresponds to $\widetilde{\Phi}(r,\theta_1)$. Then we have

$$\frac{\partial G}{\partial \theta_1} = r*(\frac{1}{n-1}\left[\frac{(r-p_R)}{bnp_R}\right]^{\frac{1}{n-1}} \theta_1^{\frac{2-n}{n-1}} [\frac{p_R}{r} - 1] - \int_{1-\widetilde{F}(p_R)}^1 \frac{(1-\mu)\mu n \widetilde{y}^{n-1}}{[(1-\mu)\theta_1 + \mu n \widetilde{y}^{n-1}]^2} dy) < 0 \tag{17}$$

since $\frac{p_R}{r} - 1 < 0$ in the region we are analyzing and the value of the integral is between 0 and 1 (hence, $-\int < 0$).

Secondly

$$\frac{\partial G}{\partial r} = \widetilde{\Phi}(r,\theta_1) - \frac{1}{n-1}\left[\frac{\theta_1}{bnp_R}\right]^{\frac{1}{n-1}} (r-p_R)^{\frac{2-n}{n-1}} (\frac{r-p_R}{r})$$

that can be positive or negative. Hence, $\left.\frac{dr}{d\theta_1}\right|_{G(r,\theta_1)=c}$ can be of either sign. Indeed, note that, when $r \to p_R$, $\frac{\partial G}{\partial r} \approx \widetilde{\Phi}(r,\theta_1) > 0$. On the other hand, as $r \to +\infty$, $\frac{\partial G}{\partial r} \to -\infty$. Since $\frac{\partial G}{\partial r}$ is continuous in r and θ_1 this implies that there is some value (θ_1, r) such that $\frac{\partial G}{\partial r} = 0$. This is $(\theta_1^D, r(\theta_1^D)$ and $\widetilde{r}(\theta_1) = c/\widetilde{\Phi}(r,\theta_1)$ is increasing up to θ_1^D and then decreasing. It should be kept in mind that, while in general the $\widetilde{r}(\theta_1)$ correspondence is backward banning at $\widetilde{r}(\theta_1^D)$, in our model the relevant reservation price cannot be larger than $p_R + k$. Hence, for some regions of parameters the relevant portion of $\widetilde{r}(\theta_1^D)$ may be only upward sloping (i.e. $\widetilde{r}(\theta_1^D) > p_R + k$). These cases can be easily observed in Figure 1 and 2.