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Abstract

Electricity demand is modeled as a time-varying parameters (TVP) vector autoegression with or without imposing cointegration. The paper applies Bayesian strategies where all or a part of the parameters are allowed to vary, and compares their forecasts performances with alternative time series models, namely a seasonal ARIMA (SARIMA) specification and a vector error correction model (VECM). Considering Italian data, the appropriate diagnostic tests and estimation results are in favour of non-stability of the parameters. However, the forecasts abilities of the models do not show significant differencies when measured by RMSE and MAE, and compared trough the Diebold Mariano statistic. On the other hand, forecast intervals of Bayesian models show higher empirical coverage rates.

Keywords. Electricity demand, forecasting, time varying coefficients model, Kalman filtering, Markov Chain Monte Carlo.
Jel codes. C11, C52, C53, Q47.

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1 Introduction

Considerable attention has been devoted to the analysis and forecast of electricity consumption by researchers and praticioners in the past several decades.

In early works, the estimation of electricity demand had been done by simultaneous structural equation models (e.g.Fisher and Kayser, 1962). Subsequently VAR and, since the papers by Engle and Granger (1987) and Engle et al. (1989), ECM models had become the standard techniques for electricity demand analyses. Further developments have relied on the use of Johansen (1988, 1995) method for estimating the long run relationship, while some attempts have adopted alternative approaches that increase the flexibility of the modelling strategies. For instance, Joutz et al. (1995) have used a Bayesian specification that allows to account for researcher's priors, while Chang and Martinez-Chombo have introduced a time varying parameters (TVP) specification to capture the evolution of the parameters over time.

Despite the big amount of studies on electricity, a much smaller number of attemps provide an explicit comparison of the forecasting performances of differnt models and none of them, at least among published works and to my knowledge, refers to the Italian market. There is therefore the need to quantify how, and to what extent, forecasts are sensible to the choice of the modeling strategy.

In the present study BVAR approaches with time varying parameters (TVP) and that may include cointegrating relations are compared with alternative time series models, namely an univariate Seasonal ARIMA (SARIMA) specification and a Vector Error Correction Model (VECM). The first model gives flexibility and exploits all available information explicitly; the second approach is appealing for its simplicitly, and third specification has become standard practice among researchers, and therefore both the last two provide natural benchmarks for comparison. By anticipating the results of this study, despite their differences, the three models do not lead to remarkable differences for forecasting aims.

These results are obtained using monthly Italian data for the period that spans from January 1990 to February 2009. A basic electricity demand equation is used, where consumption is regressed on industrial production, two series that account for calendar effects, proxies of the temperature, and eleven seasonal dummies.

Althought a demand equation is considered, prices are not included among the regressors. The main reason relies on the monthly frequency of the data and the forecasts. In the short run the demand of electricity and the possibility of switching to alternative energy sources (e.g. natural gas and distillate fuel oil) are constrained by a fixed stock of using appliances (see Silk and Jouts, 1997). Indeed, results from applied research show, on average, moderate responsiveness of electricity consumption to changes in prices (see among others: Engle et al., 1989; Filippini, 1999; and Fan and Hyndman, 2008, for a recent literature review.) Finally, an indication 'ex-post' of the validity of omitting the prices is obtained by regressing the estimated residuals from the SARIMA and VECM models on the logs of PUN baseload electricity prices[1]. The resulting coefficient does not appear significant.

The remaining part of the paper is organized as follows: the next section analyses the main features of the series and their integration properties; the models and estimations' results are presented in section 3; section 4 discusses the forcasting performance of the models, and section 5 concludes.

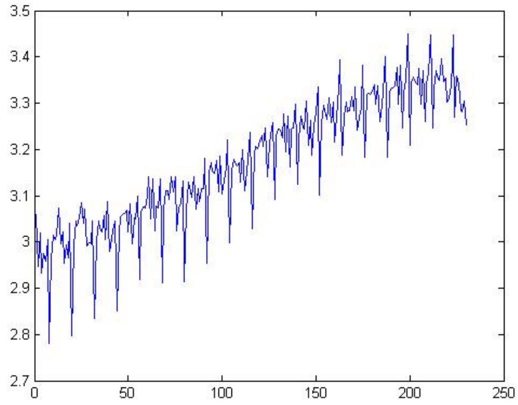
2 Data analysis and transformation

Electricity demand function is estimated using the logarithms of the electricity consumption (el). As explanatory variables, the log-transformed industrial production index (ip), cooling (CD) and heating (HD) degree days, two series that control for the calendar effect (CA) and the leap year effect (LY) are included in the models.[2] The

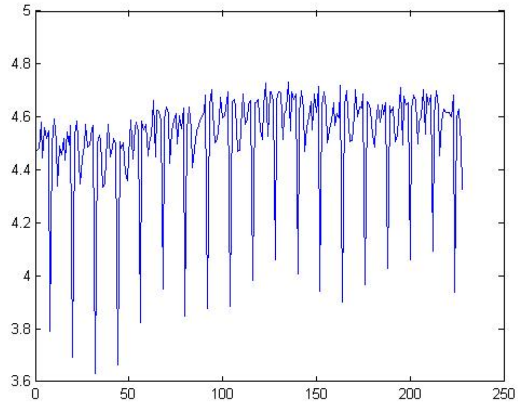
[1]Monthly averages from 2005.1 to 2009.2 are used; an MA(3) term is added to correct for residual correlation

[2]The reason for using IP (instead of alternative indicators, e.g. GDP) as income variable is primary practical: while monthly records are provided for Italian industrial production, only quarterly data are available for GDP. Second, electricity consumption is widely known to be a good predictor of GDP (in other words the causal link is from electricity to GDP), while the causal relationship between electricity and IP is in both directions.

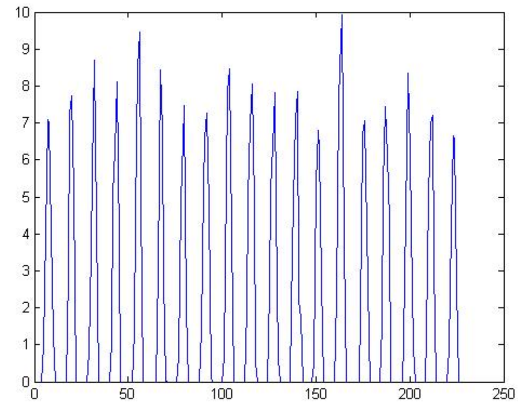
plots of the variables are reported below, while more precise definitions of the series are given in the Appendix. Lower cases stand for log-transformation of the series.



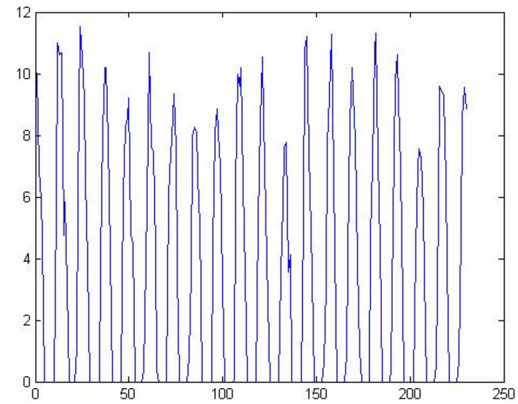
(a) Logs of electricity demand (TWh), 1990.2 - 2009.2



(b) Logs of industrial production (2000=100), 1990.2 - 2008.12



(c) Cooling degree days, 1990.2 - 2009.2



(d) Heating degree days, 1990.2 - 2009.2

Figure 1: Plot of series

As a first step of the analysis the presence of unit roots at seasonal as well as the zero frequency is detected. Among the procedures that have been developped, the one of OCSB (Osborn et al. 1988) and HEGY (Hylleberg et al. 1990) are employed here. The former allows to test the adequacy of the double filter $(1-L)(1-L^s)$; the latter tests whether $(1-L^s)$ may be preferred to one of its components. Moreover, to investigate whether these filters lead to improved forecasts, the predictive performance of univariate models based on alternative transformations of the series is considered.

The OCSB method considers the auxiliary regression:

$$\phi_p(L)\,\Delta_1\Delta_{12}y_t = \sum_{s=1}^{12}\delta_s d_{s,t} + \pi_1\Delta_{12}y_{t-1} + \pi_2\Delta_1 y_{t-12} + \epsilon_t \qquad (1)$$

where the order p of the polinomial $\phi_p(L)$ is chosen such that the estimated residuals are approximately white noise. It holds that if $\pi_2 = 0$ the filter Δ_{12} is appropriate, and if $\pi_1 = \pi_2 = 0$ the double filter $\Delta_1\Delta_{12}$ is needed. Table 1 shows the estimates of (1).

Variable	lags	$t(\pi_2)$	$F(\pi_1, \pi_2)$
el	$1, 12$	-6.895^{**}	23.779^{**}
ip	$1, 2, 5, 10$	-3.232	6.903

Table 1: **OCSB method**; $**$ denotes significance at the .05 level; 'lags' refers to the lagged $\Delta_1\Delta_{12}$ variables included in the auxiliary regression

The second approach involves the HEGY regression, which in case of monthly data is (see Franses, 1991):

$$\begin{aligned}\phi(L)\,\Delta_{12}y_t = {} & \mu + \sum_{i=1}^{11}\gamma_i D_{it} + t_t + \psi_1 y1_{t-1} + \psi_2 y2_{t-1} + \psi_3 y3_{t-1} + \\ & + \psi_4 y3_{t-2} + \psi_5 y4_{t-1} + \psi_6 y4_{t-2} + \psi_7 y5_{t-1} + \psi_8 y5_{t-2} + \\ & + \psi_9 y6_{t-1} + \psi_{10} y6_{t-2} + +\psi_{11} y7_{t-1} + \psi_{12} y7_{t-2}\end{aligned} \qquad (2)$$

where the auxiliary regressors are appropriately defined as in Franses (1998).
The component hypothesys $\psi_1 = 0$, $\psi_2 = 0$, $\psi_3 = \psi_4 = 0$, $\psi_5 = \psi_6 = 0$, $\psi_7 = \psi_8 = 0$, $\psi_9 = \psi_{10} = 0$, $\psi_{11} = \psi_{12} = 0$ correspond to separate tests for the unit roots contained in the real valued $(1 - L)$, $(1 + L)$, $(1 + L^2)$, $(1 + L + L^2)$,$(1 - L + L^2)$, $\left(1 + 3^{1/2}L + L^2\right)$, $\left(1 - 3^{1/2}L + L^2\right)$ respectively.
The results of the test performed on the series el and ip are reported in Tables 2 and 3.

Test	t-stat	F-stat
t_1	-1.472	
t_2	-2.427*	
w_1		1.240
w_2		5.259*
w_3		2.448
w_4		4.982*
w_5		1.634

Table 2: **HEGY method**; variable *el*; no trend; $*$ and $**$ denote significance at the .10 and .05 level, respectively; critical values are reported in Franses(1991)

Test	t-stat	F-stat
t_1	-2.963*	
t_2	-2.434*	
w_1		10.270**
w_2		6.911**
w_3		1.551
w_4		11.224**
w_5		1.523

Table 3: **HEGY method**; variable *ip*; trend; $*$ and $**$ denote significance at the .10 and .05 level, respectively; critical values are reported in Franses(1991)

The results of the OCSB and HEGY tests, reported in Tables 1 - 3, suggest conflicting interpretations of the type of seasonality in the series. To solve this apparent conflict Table 4 and Table 5 report the one-step ahead and multi-step ahead forecasts of the series. To evaluate the set of forecasts the observations from 1990m1 to 2003m12 are used for the estimation and forecasts are generated for the sample 2004m1-1204m12. The reason for choosing this sample is that the years next to 2004 registered temperatures abnormally high or low. The RMSE is the evaluation criterion. Only models that passed the tests on residuals autocorrelation are reported. For the series el, the one-step ahead RMSE is smaller for the $ARMA\,(1,12;1)$ model on the untrasformed series; while for the multi-step ahead the same model on the differences of el has the smallest RMSE. In contrast with the results for el, considering the series ip the model for the variable transformed according to the HEGY test outperforms the other models on both, one-step and multi-step ahead.

In sum, it may be concluded that a small number of imposed unit roots leads to better forecasts for the variable el. Therefore, the $(1-L)$ filter is used for this variable in the remaining of the paper. As for the variable ip the forecast evaluation and the HEGY test (in contrast with the OCSB method) provide evidence of some seasonal unit roots. In particular, according to the results of the HEGY test, ip should be substituted by $ip^* = \left(1-3^{1/2}L+L^2\right)\left(1-L+L^2\right)ip$ before analysing the cointegration between el and ip^*. However, this filter does not seem to lead to good results in our particular case, and it appears more opportune to treat this variable as integrated of order one at the zero frequency only.

Filter	Levels	Δ_1	Δ_{12}	$\Delta_1\Delta_{12}$
Model	$arma\,(1,12;1)$	$arma\,(1,12;1)$	$arma\,(2,1)$	$ma\,(1,12,13)$
Determ.	t,d	d	-	-
$1-step$	.361	.390	.625	.392
$h-step$	.375	.353	.726	.370

Table 4: Univariate models for **electricity demand RMSE for 2004.1 - 2004.12**; for each filter the reported specification is the one that minimizes the BIC among those with not significant LM of order 2; all RMSE refer to the original electricity demand series

Variable	Levels	Δ_1	Δ_1	$\Delta_1\Delta_{12}$
Model	$ar\,(1,12)$	$arma\,(1,12;1)$	$ma\,(1,12,13)$	$ma\,(1-5)$
Determ.	t,d	d	-	-
$1-step$	3.342	3.205	4.223	2.526
$h-step$	3.601	2.377	3.817	2.051

Table 5: Univariate models for **industrial production RMSE for 2004.1 - 2004.12**; for each filter the reported specification is the one that minimizes the BIC among those with not significant LM of order 2; all RMSE refer to the original industrial production series

3 Modeling strategies

3.1 Preliminary methods

3.1.1 Univariate analysis

In the previous section it has been shown that the ARIMA($p = 1, 12; q = 1$) specification outperforms for modeling electricity consumption all other univariate model. Estimation results are reported in Table 6.

3.1.2 Fixed coefficient VECM

Having assesed that series present unit roots, in this section the existence of cointegration is checked for. In particular, the series of the electricity demand and the industrial production, which appear to be integrated at the zero frequency, may show non-seasonal cointegration.

Adopting the method proposed in Johansen (1988), the starting point of the cointegration analysis is a VAR specification for the $nx1$ vector of $I(1)$ variables X_t:

$$X_t = A_1 X_{t-1} + \ldots + A_p X_{t-p} + \Psi D_t + u_t \tag{3}$$

where D_t contains deterministic components, and u_t, is an $nx1$ i.i.d. Gaussian error

$Variable$	$Coefficient$	$StdError$
C	-0.044**	0.006
$\Delta el(-1)$	-0.446**	0.057
$\Delta el(-12)$	0.108**	0.042
$D1$	0.023**	0.005
$D2$	-0.053**	0.007
$D3$	0.024**	0.007
$D4$	-0.030**	0.010
$D5$	0.026**	0.013
$D6$	0.011	0.015
$D7$	0.004	0.017
$D8$	-0.218**	0.021
$D9$	0.022**	0.017
$D10$	0.119**	0.015
$D11$	0.017**	0.007
RU	0.002**	0.000
LY	0.043**	0.008
HD	0.006**	0.001
CD	0.013**	0.002
$MA(1)$	0.488**	0.085
$R2 = 0.973$	$LM(4) = 1.747$	$LM(5) = 1.4$

Table 6: **SARIMA model** - Estimation results; $*$ and $**$ denote significance at the .10 and .05 level, respectively

vector. Equation (3) can be reparametrized as:

$$\Delta X_t = \Pi X_{t-1} + \Pi_1 \Delta X_{t-1} + \ldots + \Pi_{p-1} \Delta X_{t-p+1} + \Psi D_t + u_t \tag{4}$$

where $\Pi = -(I_n - A_1 - \ldots - A_p)$, $\Pi_i = -(I_n - A_1 - \ldots - A_i)$, $i = 1, \ldots, p-1$, which is the VECM representation of the original VAR system (see, among others, Charemza and Deadman, 1992). If cointegration among the variables X_t is present, model (4) includes both long-run and short-run stationary components. The maximum likelihood method by Johansen tests the presence of cointegration at the systems level by determining the rank of the long-run matrix, Π. If $rank\Pi = r$, with $0 < r < n$, the matrix Π can be decomposed as $\Pi = \alpha\beta'$, where α, is an nxr matrix of adjustment parameters and $\beta$is an nxr matrix containing the r cointegrating relations among the variables in X_{t-1}. The Johansen approach enables to estimate the parameters β, and to assess the number of $I(0)$ linear combinations among the X_t variables.

In the present case, X_t consists in the logged electricity demand and industrial production, while D_t includes a constant, the series that account for calendar and temperature effects and seasonal dummies. As suggested in Johansen (1995), the dummies are orthogonalized on the constant $1/12$, in such a way that they do not generate a trending term. Since one cointegrating relationship is found among the variables, this is included in the model that can be written in the VECM form[3]. The estimation results related to electricity demand equation for the whole sample (1990.1 - 2008.12) are reported in the Table 7. Based on diagnostic checks, the estimated specification appears satisfactory.

3.1.3 Stability analysis

Over the last seventeen years the response of electricity consumption to its determinants may have changed in several ways (see Bertoldi and Atanasiu, 2007). For example, it is possible that an increase in summer temperatures (captured by the series of CD) has a larger impact now than in 1990, and this could be due to the diffusion of cooling appliances or, possibily, a changement in people's utility function. Alternatively, the adgiustement to the long-run equilibrium may have varied over time, or it could be the case for other factors. There are different types of tests for parameters

[3]The same variables appear in both equations. HD and CD shouldn't be very helpful to predict industrial production. However, the estimated coefficients are so small, that they may imply little distorsion.

Variable	Coefficient	StdError	li
$ect(-1)$	-0.218**	0.062	0.149
$\Delta el(-1)$	0.222**	0.073	0.124
$\Delta ip(-1)$	-0.194**	0.031	0.052
$\Delta el(-2)$	0.223**	0.066	0.036
$\Delta ip(-2)$	-0.141**	0.029	0.018
$MA12el(-1)$	-0.395**	0.063	0.059
$MA12ip(-1)$	0.081**	0.030	0.191
$d1$	0.016**	0.007	0.046
$d2$	-0.071**	0.008	0.19
$d3$	0.081**	0.009	0.041
$d4$	-0.041**	0.011	0.091
$d5$	0.053**	0.012	0.1
$d6$	-0.010	0.016	0.205
$d7$	-0.004	0.018	1.051
$d8$	-0.273**	0.019	0.990
$d9$	0.049**	0.022	0.14
$d10$	0.093**	0.022	0.433
$d11$	0.067**	0.019	0.722
ru	0.002**	0.000	0.223
ly	0.035**	0.008	0.176
hd	0.004**	0.001	0.783
cd	0.013**	0.002	1.336
c	-0.031**	0.006	0.255
	$R2 = 0.973$	$LM(5) = 3.178$	$LC = 8.235$

Table 7: **VECM** Estimation results for electricity demand equation; li and LC stand for, respectivey, the single coefficient and the cumulative results of Nyblom statistic; $*$ and $**$ denote significance at the .10 and .05 level, respectively

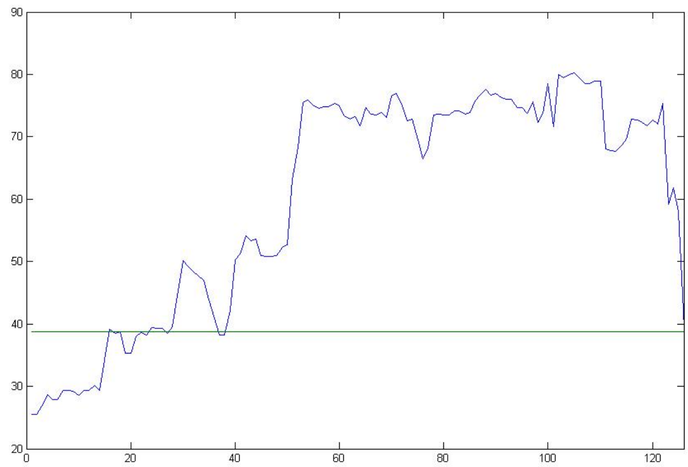


Figure 2: Chow test sequence and relevant Andrews critical value

stability (see for example Marcellino, 2002). Here, the methods of Quandt (1960) and Nyblom (1989) are adopted. The first tests the hypothesis of parameters stability (of the demand equation) against the alternative of a single break at unknown date. In particular, the method of Quandt (1960) considers the maximum value of the Chow test computed recursively for every possible breakdate[4]. The tests' sequence is reported in Figure (3.1.3) together with the opportune critical value (Andrews, 1993). As it appears from the graph, the sequence of Chow test lies above Andrew's critical value at several dates, suggesting instability of the parameters. Indeed, as it is possible that this conclusion is distorted by the presence of seasonal dummies in the equation, the test is repeated over series previously adjustded for seasonality. The results still reject the null, and thus confirm the ropture with the hypothesis of parameters' constancy. Second, to better assess the nature of the instability a further method (Nyblom 1989, Hansen 1992) that allows for breaks at unknown dates as well as random walks parameters is adopted.

Since the interest is in the dynamics of the VECM, the test is applied to the coefficients of I(0) variables. In practice since the model is unrestricted and it includes exogenous variables, it is estimated through a two-step procedure (1st step: cointegrating eq. Jo-

[4]As suggested in Hansen (2001) the top and bottom .15 of dates' series is discarded

hansen; 2nd step OLS). The second stages are re-estimated and Nyblom tests are performed. The results for the electricity equation are shown in Table 7. According to the above figures, the joint statistic rejects the null of stability at the 1 percent level. Single-coefficients tests show instability due to the impact of Summer months (june-october), HDD and CDD. The remaining variables appear more robust over the sample.

3.2 TVP - BVARs with or without cointegration

As above seen, the results of the stability tests suggest that coefficients may vary over time. Here, (4) is replaced by:

$$\Delta Y_t = B_t X_t + E_t \tag{5}$$

$$B_t = [A_{1t}, \Omega_t] \tag{6}$$

$$X_t = \left(\Delta Y_{t-1}^{'}, Z_t\right)^{'} \tag{7}$$

where the evolution path of the parameters is defined as:

$$\tilde{\beta}_t = F\tilde{\beta}_{t-1} + \eta_t \tag{8}$$

$$\tilde{\beta}_t = \beta_t - \bar{\beta}_0 \tag{9}$$

$$\beta_t = vec\left(B_t^{'}\right) \tag{10}$$

The regressors' vector X_t has size $(kX1)$, where k is given by the number of equations times lags plus the rows' number of Z_t. The adopted approach is to treat (5) as a simple VAR model for ΔY_t, possibly augmented with the inclusion of the disequilibrium term (previously estimated using classical techniques) as an additional regressor in a Bayesian VAR framework (see Alvarez and Ballabriga, 1995; and Amisano and Serati, 1999). The vector Z_t includes the seasonal dummies, the correctors for calendar effects, the series HD and CD and may include the cointegrating vector.

Equations (5) and (8) consitute the measurement and state equations of a state-space representation, wich is the standard framework for estimating TVP models.
The error terms of the m observed y_t and the K unobserved β_t components are assumed i.i.d. normal distributed, with $E\left(\eta_t \epsilon_t^{'}\right) = 0$[5]. The parameters evolve as a random walk. The prior for the initial state of the time-varying coefficients is Normal. The inverse variance-covariance matrices of both, the measurement and the state, equations are assumed to follow the Wishart distribution (conjugate priors). The matrix F in (8) is set to be the identity matrix, that is the parameters follow a random walk[6].
The sample is split in two parts. The first set of observations are used to calibrate the parameters of the prior distributions. In particular, the mean and the variance of B_0 are chosen to be the OLS point estimates on the initial subsample and their variances. The degrees of freedom, ν_n and ν_β, of the Wishart distributions are set to be, respectively, 6 and 100 plus the dimension of each matrix[7]. The parameter ν_β is chosen in such a way to shrink the distribution of the parameters. The scale matrices are chosen to be diagonal matrices, labeled $S_n = k_n I_t$ and $S_\beta = k_\beta I_z$ for the precision matrices of the measurement and state equations. Table 8 summarize the hyperparameter of the model.

The posterior distributions of the parameters are obtained by performing the Kalman filter (forward recursion) and the smoothing techniques of Carter and Kohn (1994) (backward recursion that allows to reconstruct the in sample evolution path of the β_s by using the complete set of the information). The final estimates of the states for the electricity demand equation are reported in table 9 and table10. Selected time-varying coefficients are reported in Figures 3 and 4.

[5]Here the var-cov matrix is assumed to be block-diagonal. Refer among others to Cogley and Sargent (2001) and Amisano and Federico (2004) for examples of non block diagonal forms.

[6]Note that in model (5) - (8) the only source of variability are model's coefficients, while the variance and covariance matrix of the shocks is assumed constant over time (see Primiceri, 2005; and Cogley et al. 2008 for a different approach on this point).

[7]The degrees of freedom exceed the dimension of the Wishart for both, the measurement and the state, equations and therefore the inverse Wishart are proper.

μ_0	μ_{OLS}
Σ_0	Σ_{OLS}
ν_n	m+6
ν_β	K+100
k_n	10
k_β	10^6

Table 8: Hyperparameters

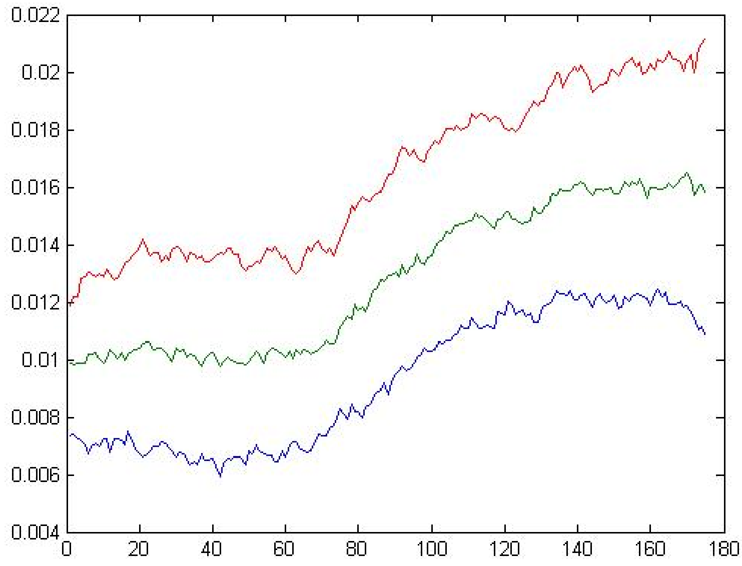


Figure 3: Evolution path of Cooling degree days'(CD) impact on electricity demand growth. TVP BEC model

Variable	Coefficient	Std.Error
$ect(-1)$	-0.495**	0.126
$\Delta el(-1)$	-0.012	0.167
$\Delta ip(-1)$	-0.148**	0.066
$\Delta el(-2)$	0.129	0.124
$\Delta ip(-2)$	-0.087*	0.048
$MA12el(-1)$	-0.178*	0.105
$MA12ip(-1)$	0.030	0.037
$d1$	0.031**	0.013
$d2$	-0.040**	0.015
$d3$	0.075**	0.016
$d4$	-0.006	0.019
$d5$	0.062**	0.016
$d6$	0.024	0.023
$d7$	0.035	0.023
$d8$	-0.213**	0.031
$d9$	0.082**	0.045
$d10$	0.176**	0.044
$d11$	0.084**	0.037
ru	0.001	0.004
ly	0.036**	0.013
hd	0.008**	0.004
cd	0.015**	0.006
$const$	-0.050**	0.012

Table 9: **BECM** Final estimates of the states, B_T, and square roots of the corresponding variances for the electricity demand equation; $*$ and $**$ denote significance at the .10 and .05 level, respectively

Variable	Coefficient	Std.Error
$\Delta el(-1)$	-0.207	0.198
$\Delta ip(-1)$	-0.096*	0.054
$\Delta el(-2)$	-0.003	0.147
$\Delta ip(-2)$	-0.022**	0.011
$MA12el(-1)$	-0.263**	0.131
$MA12ip(-1)$	0.021	0.051
$d1$	0.017	0.021
$d2$	-0.067**	0.024
$d3$	0.054**	0.025
$d4$	-0.047	0.031
$d5$	0.055*	0.030
$d6$	0.016	0.037
$d7$	0.013	0.035
$d8$	-0.248*	0.044
$d9$	0.083	0.054
$d10$	0.124**	0.054
$d11$	0.047	0.045
ru	0.002	0.005
ly	0.005	0.026
hd	0.010**	0.005
cd	0.018**	0.009
c	-0.070**	0.023

Table 10: **BVAR** Final estimates of the states, B_T, and square roots of the corresponding variances for the electricity demand equation; $*$ and $**$ denote significance at the .10 and .05 level, respectively

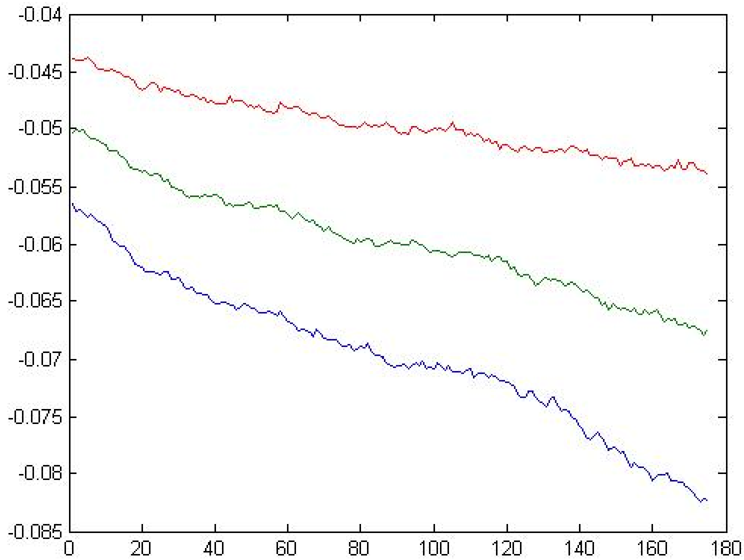


Figure 4: Evolution path of the constant term in the electricity consumption eq of the TVP BECM

3.2.1 BVAR models with coefficients partly varying and partly constant

The need to estimate a large number of parameters can worsen the performance (particularly out-of-sample) of TVP-BVAR models in some empirical applications. In order to reduce the dimension of paremeters space, equation (5) can be replaced by:

$$\Delta Y_t = B_t X_t + \Gamma W_t + \epsilon_t \tag{11}$$

where impacts of the variables in W_t are assuned constant over time.[8] In practice, only the coefficients with hightest evidence of instability are ellowed to vary: i.e. X_t includes the lagged dependent variables, $D7$, $D8$, the HD and CD, and the adjustements to the equilibrium term. As in the unrestricted case, (11) is estimated through the multi-move Gibbs sampling technique, a particular variant of MCMC algorithm that allows to draw from the conditional posterior distribution instead of the high dimensional

[8] Alternatively, estimation strategies proposed to tighten the dimension of parameters matrix could be used (see among others, Canova and Ciccarelli, 2004; and Canova, 2007; and Sims et al., 2006).

joint posterior of the parameters. In the case in which the parameters are partly constant and partly varying, Gibbs sampling is carried out in four steps, sampling firstly from the posterior of the time-varying parameter, B_t, and in turn from the posterior of the fixed coefficients, Γ, and finally of the precision matrices, H^{ϵ} and H^{η}, conditional on the observed data and the rest of parameters. The final estimates are silmilar to those reported in Table 9 and Table 10, and therefore are not reported.

4 Predictive ability comparison

The models presented in the previous section are now compared based on their predictive ability. The performances are measured by the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) computed over recursive samples. Relative comparison of forecasts is based on the Diebold Mariano statistic. [9]

The forecasts are made as follows: the first set of forecasts are based on the models estimated through data biginning in January 1990 and ending in December 1999. Using this sample, dynamic predictions are made for the following twelve months. Then the estimation period is extended up to January 2000, and predictions are generated for the next months up to January 2001. This process of adding one year of observatios, re-estimating, and forecasting up to twelve months ahead is repeated until January 2008 has been added to the estimation period .

Using this set forecasts, prediction errors are then computed and evaluated in two ways. In tables 11 and 12 the performance of one-month, two-month up to twelve-month ahead predictions is reported. Looking at tables' results, imposing a bayesian prior on the parameters and of allowing them (or a few of them) to vary over time does not seem to lead to a remarkable advantages.[10]

From the figures in Tables 11 and 12 it appears that for one up to two months ahead the univariate model has the largest forecasting accuracy. As the number of forecasted

[9] The parameters of the TVP-BVAR are considered constant for the forecasting period.

[10] These conclusion are not general; however the latter result is somehow supportive of the evidence found by Joutz et al.(1995) using fixed coefficients BVAR for USA data.

months increases the SARIMA performs badly, whereas forecasts by VECM and by TVP BVAR models are similar, but slightly more accurate in case of VECM. This difference in accuracy diminishes for ten-month up to twelve-month forecasts. To restrict some of the parameters of the TVP BVAR model to be constant over time does not improve model's performance; in fact this leads to slightly worse forecasts.

To conclude about the relative comparison of forecast accuracies the Diebold Mariano statistic is used. Being *di* the loss function associated to model *i* the DM statistic assesses whether the loss differential between the two competitive models differs from zero. The test is given by:

$$S = d / \left(2 f_d(0)/T\right)^{.5} \tag{12}$$

where d is the average of loss function differentials and $f_d(0)$ is an estimate of loss differentials' asymptotic variance. The comparison of the VECM vs the BECM and the VECM vs the BVAR model based on the DM statistic are presented in Table 13. The Figures are obtained when the loss function is the RMSE; when the MAE is used the results are very similar and then are not reported.

As expected, the results do not lead to the rejection, at conventional levels, of the hypothesis of equal prediction errors.

Tables 11-13 provide absolute and relative measurements of forecasts accuracy that are based on a large number of repetitions and statistically meaningful. However, economic losses due to various forecasts errors are could not be easy interpretable by comparing statistical measures (e.g. RMSE) or through statistical tests. To give a more immediate view of the costs/benefits associates to the various models, the forecasts in GWh and errors as percentages of the actual values for the years 2007-2009 [11] are reported in Table 14.

[11] Yearly forecasts are obtained as sum of monthly figures.

m	SARIMA	VECM	TVP-BECM	TVP-BVAR	PartTVP-BECM	PartTVP-BVAR
1	0.009	0.017	0.026	0.017	0.030	0.017
2	0.015	0.021	0.022	0.021	0.031	0.024
3	0.024	0.023	0.028	0.024	0.037	0.026
4	0.018	0.023	0.032	0.028	0.040	0.029
5	0.030	0.022	0.039	0.032	0.041	0.033
6	0.035	0.024	0.033	0.034	0.041	0.032
7	0.041	0.024	0.041	0.038	0.053	0.037
8	0.051	0.025	0.043	0.039	0.052	0.038
9	0.054	0.028	0.047	0.039	0.053	0.042
10	0.051	0.029	0.043	0.039	0.059	0.043
11	0.061	0.031	0.052	0.039	0.061	0.043
12	0.066	0.033	0.043	0.037	0.054	0.041

Table 11: **RMSE** for electricity demand - Dynamic forecasts over the period $I : 2000.1 - 2000.12 ... XCVIII : 2008.2 - 2009.2$, when the parameter are estimated for $I : 1990.1 - 1999.12 ... XCVIII : 1990.1 - 2008.1$

m	SARIMA	VECM	TVP-BECM	TVP-BVAR	PartTVP-BECM	PartTVP-BVAR
1	0.008	0.014	0.019	0.013	0.021	0.013
2	0.012	0.016	0.016	0.016	0.024	0.018
3	0.018	0.019	0.021	0.020	0.029	0.021
4	0.017	0.019	0.026	0.022	0.032	0.023
5	0.027	0.019	0.030	0.026	0.032	0.027
6	0.032	0.020	0.025	0.028	0.034	0.026
7	0.038	0.020	0.035	0.031	0.043	0.028
8	0.049	0.020	0.037	0.032	0.040	0.033
9	0.053	0.021	0.039	0.032	0.041	0.035
10	0.045	0.022	0.036	0.032	0.049	0.036
11	0.049	0.023	0.045	0.032	0.050	0.036
12	0.052	0.023	0.037	0.030	0.043	0.033

Table 12: **MAE** for electricity demand- see tab:RMSE

Now I consider the last three years and calculate what forecast errors would be made by using the various models to forecast in December the demand over the next year. The results show that the VTP-BECM, which allows for both evolving parameters and adjustment toward the long-run lead level, leads to better economic decisions. For instance, forecasting the demand in 2008 through the TVP-BECM instead of the VECM would eliminate 90% of the error with relevant consequences (e.g. for plant management, stock policy, price policies and budget preparation and control). Moreover, the difference in the errors gets larger for 2008 and 2009 because more flexibility allows to better capture the changes in the economic scenario. However, none of the models is able to capture the deep shift of demand in 2009

In what precedes point forecasts have been the object of the analysis. Additional interesting information can be gained from the evaluation of forecast intervals and corresponding empirical coverage rates. A series of 90% forecast intervals (5% and 95% forecast quantiles) are calculated using recursive samples in the same fashion as above

m	VECM vs TVP-BVAR	VECM vs TVP-BECM	VECM vs PartTVP-BVAR
1	0.427	-0.920	-0.728
2	0.211	0.013	-1.717
3	-0.081	-0.538	-1.372
4	-1.254	-1.169	-1.752
5	-2.409	-1.769	-5.054
6	-1.505	-1.083	-1.296
7	-2.582	-2.792	-1.959
8	-2.313	-3.453	-5.956
9	-1.619	-3.631	-3.332
10	-1.750	-2.912	-1.330
11	-1.327	-6.263	-1.523
12	-1.097	-2.471	-1.425

Table 13: **DM** for electricity demand when the loss function is the RMSE - see tab:RMSE

	VECM	TVP-BECM	TVP-BVAR	PartTVP-BECM	PartTVP-BVAR
2007					
Forecast	344.528	335.750	332.792	331.846	333.548
Prc error	1.4%	-1.2%	-2.1%	-2.4%	-1.9%
2008					
Forecast	347.461	338.700	337.041	340.718	349.871
Prc error	2.4%	-0.2%	-0.7%	0.4%	3.1%
2009					
Forecast	337.598	331.263	334.770	334.512	341.244
Prc error	6.5%	4.5%	5.7%	5.6%	7.7%

Table 14: Demand forecasts for 2007.1:2007.12, 2008.1:2008.12 and 2008.1:2008.12 (TWh), and corresponding percentage errors

	VECM	TVP-BECM	TVP-BVAR	PartTVP-BECM	PartTVP-BVAR
n=1	81%	88%	95%	87%	95%

Table 15: Empirical coverage rates electricity demand growth 90% forecast intervals

described. Then the frequency at which the actual growth rates of demand are contained in the forecast intervals is calculated. In case of the VECM, the distribution of the errors and therefore forecast intervals are obtained by performing the bootstrap over the residual sample; for the Bayesian models they can be derived from the posterior simulation of parameters and variances. In all cases, TVP Bayesian models have higher coverage rates than the classical VECM. The reason relies on the fact that bayesian forecast intervals intrinsecally incorporate parameters' uncertanty. In particular, BECMs with all or part of the coefficients varying display coverage rates close to the desired 90%; TVP-BVARs are somehow overcovering, while forecast intervals of the VECM contain the actual growth rates of demand 81% of times only (Table 15).

5 Conclusion

This paper analyses alternative models for forecasting electricity demand. These are a SARIMA specification, a VECM and TVP BVAR models that may or may not include the cointegrating vector among the regressors. The latter specifications seem very appealing, as i) they use all the researcher's information about the coefficients, and ii) they account for possible changes in the parameters over time. Stability analyses show that paramenters vary over time and that they evolve as random walks. Despite this evidence, TVP VARs/TVP BECMs and VECMs show similar forecasting perfomances (as measured by RMSE and MAE). Indeed, to restrict some of the coefficients to be constant over the sample does not improve out-of-sample results. The same evidence is reached when evaluating models' relative forecasting performances by the Diebold-Mariano statistic. In particular, let alone the SARIMA model (which could be preferable for 1-2-step ahead forecasts only) it is not possible to find a model that works clearly better than the others at small, intermediate or large horizons. However, the fixed coefficient VECM performs slightly better for 3-month up to 12-month ahead forecasts, but the differential between the VECM and the TVP-BVAR model tends to reduce for 10-12 step ahead forecasts. For further research, I plan $a)$ to extend the sources of time variability to the variance-covariance matrices of the shocks, and $b)$ to apply a strategy alternative to the one already adopted in the present study to tighten parameters' dimension.

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6 Appendix

Datails of the examined series. Electricity demand data are provided by Terna. Series of the Italian industrial production are published monthly by ISTAT. Also, the two vectors accounting for calendar effects are by ISTAT. The source of the data on Heating and Cooling Degree Days is Bloomberg, and the two series are defined as:

$$CDD = max\,(0, t - 18)$$

$$HDD = max\,(0, 18 - t)$$

where t is the average daily temperature.