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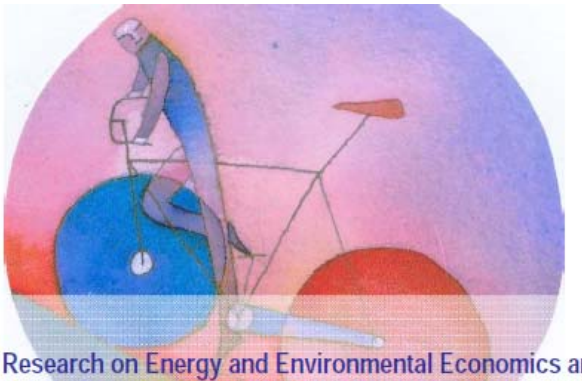

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UNCERTAINTY IN INTEGRATED ASSESSMENT MODELLING: CAN GLOBAL SENSITIVITY ANALYSIS BE OF HELP?

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Abstract. The complexity of integrated assessment models (IAMs) prevents the direct appreciation of the impact of uncertainty on the model predictions. However, for a full understanding and corroboration of model results, analysts might be willing, and ought to identify the model inputs that influence the model results the most (key drivers), the direction of change associated with the variation of a given input and the overall model structure (interaction analysis). We show that such information is already contained in the data set produced by Monte Carlo simulations commonly used in IAM studies and that can be extracted from it without additional calculations. Our discussion is guided by an application of the proposed methodologies to the well-known DICE model of William Nordhaus (2008). A comparison of the proposed methodology to approaches previously applied on the same model shows that robust insights concerning the dependence of future atmospheric temperature, global emissions and current carbon costs and taxes on the model's exogenous inputs can be obtained. The method avoids the fallacy of a priori deeming the important factors based on the sole intuition.

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1 INTRODUCTION

Climate change is a very complex phenomenon with global consequences reaching far into the future. As such, climate change (CC henceforth) is characterized by ample uncertainties as to its causes as well as its impacts. Uncertainty permeates the economic evaluation of damages and of mitigation policies. Economists use integrated assessment models (IAMs) to perform long term analysis of the economic effects of climate change. This practice is not free from criticism (see Tol, 2003; Weitzman, 2009); yet, scientific models are today's intermediaries between science and policy. When environmental and climate change issues are considered, IAMs play a central role in aiding policy makers during the formulation of mitigating strategies and risk management plans. These plans are very complicated machines due to the intricacy of the phenomena under investigation, their space and time scales and the variety of features they capture, ranging from physical laws to socio-economic aspects. This makes it impossible to have a direct understanding of the relationship between the endogenous and exogenous variables. Climate scientists and decision-makers are then exposed to the risk of drawing conclusions without a full appreciation of the model's behavior and of the most critical assumptions influencing the outcomes. This generates an issue of trust in model results (Risbey et al., 2005). Furthermore, uncertainties generated by our partial understanding of the laws governing the social-economic-environmental phenomena under investigation, by their intrinsic variability or by the lack of data, characterize both the model building and the result interpretation phases (Apostolakis, 1990; Bernstein et al., 2009; Webster, 2009). In these circumstances it is even more important that "the standard of quality for models must be high, lest model use falls into disrepute and stakeholders reject the use of models altogether" (Saltelli and D'Hombres, 2010, p. 302). The problem is perceived in the climate change community (Oppenheimer et al., 2007): for instance, Bernstein et al. (2009) underline that "dealing consistently with risk and uncertainty across the Intergovernmental Panel on Climate Change (IPCC) reports is a difficult challenge" (p.3) and that "observed differences in handling uncertainties by the three IPCC working groups emerge" (p.1).

How can we overcome these problems when using climate-economy IAMs? Webster (2009) suggests that, independently of the IPCC working group affinity, it is appropriate for the community to produce more instances of rigorous analysis of uncertainty for their respective models and projections. The US Environmental Protection Agency, for instance, recommends that model developers and users perform sensitivity and uncertainty analysis to help determine when a model can be appropriately used to inform a decision (US EPA, 2009). However, although sensitivity analysis (SA) techniques are the key ingredient needed to draw out the maximum capabilities of mathematical modeling (Rabitz, 1989), surveys show that the application of the most recently developed methods is quite limited in the field of climate change economics. Saltelli and Annoni (2010) review several papers published in prominent scientific journals such as *Science* and *Nature* and conclude that the most widely utilized methods are one-factor-at-a-time (OFAT) techniques. Generally defined, OFAT are methods of

designing experiments involving the testing of factors, or causes, one at a time instead of all factors simultaneously. These methods are quite inadequate for identifying the factors on which to focus scientists' or decision-makers' attention in the presence of uncertainty; furthermore, they undermine the analysts' ability of understanding the model behavior.

The task is however challenging. On the one hand, one needs methods that minimize the computational burden due to the numerical complexity of IAMS. To avoid pitfalls in delivering information to decision-makers, these methods must be robust and take all sources of uncertainty into account.

In this paper we propose a methodology based on a set of recent advances in the areas of high-dimensional model representations and of global sensitivity analysis. Our goal is to demonstrate that insights concerning direction of change, model structure and key uncertainty drivers can be directly extracted from the sample generated by a traditional Monte Carlo uncertainty propagation procedure, without the need of ad-hoc sampling plans. Moreover, the computational cost is minimized. Direction of change is here meant in a global sense, as opposed to the traditional local information of comparative statics. The key, in this respect, is to base the analysis on the high-dimensional model representation (HDMR) theory of Sobol' (1993) and Rabitz and Alis (1999). HDMR also grants understanding of whether the endogenous variable response to changes in the exogenous variables is equal to the superimposition of their individual effects or whether interactions are relevant (model structure). The methodology is then complemented by the use of density-based methods for the identification of key uncertainty drivers in the presence of both correlated and uncorrelated exogenous variables.

Numerical experiments are performed using one of the best known IAMs, Nordhaus' DICE model. The results show that a systematic application of these methods provides several crucial insights to both analysts and policy-makers. Furthermore, one avoids pitfalls in the identification of the variables and areas on which to focus additional information collection and/or modeling efforts.

The remainder of the paper is organized as follows. Section 2 reviews the existing literature and provides a brief snapshot of how global sensitivity analysis methods are, or are not, being used. Section 3 presents our proposed methodology, whose estimation and computation aspects are considered in Section 4. The global SA is applied on the DICE and the results of this exercise are presented in Section 5. Concluding remarks close the paper.

2 GLOBAL SENSITIVITY ANALYSIS METHODS IN IAMS: A CURSORY LITERATURE REVIEW

There are many competing IAMs that are being used to analyze the role of uncertainty in the economics of climate change. The publication of the Stern Review (Stern, 2007) re-ignited a debate

on the performance of IAMs under different parameterizations and the policy conclusions that can be drawn from the results. A quick web search using terms such as "climate change, uncertainty and sensitivity" would likely bring forth the series of rebuttals from Dietz et al. (2007a, 2007b), Nordhaus (2007a, 2007b), Stern and Taylor (2007) and Tol and Yohe (2006, 2007). Weitzman (2007) focused on the sensitivity of model outputs primarily to the choice of the discount rate and to a few other selected model inputs. However, a literature review shows that the same critical scrutiny is rarely performed on the quantitative performance of different IAMs and the robustness of their formulation.

IAMs are becoming increasingly used as a tool to study how uncertainty and ambiguity affect policy makers' decisions regarding climate change. Golub et al. (2011) provide a comprehensive overview of different approaches used to model uncertainty when applying IAMs. Millner et al. (2010), Lemoine and Traeger (2011), and Iverson and Perrings (2012) are recent examples of applications using the DICE model to study these areas of decision science.

Monte Carlo simulation to propagate uncertainty in model inputs is becoming part of best practices in the IAM literature. It has been used for an uncertainty analysis of the DICE model (Nordhaus, 1994, 2008) and in different vintages of the PAGE model (Hope, 2006). It is employed in a recent study by Dietz (2011) in an assessment of catastrophic climate change based on the PAGE model and by Nordhaus and Popp (1997) using DICE and Popp (2004) using ENTICE, an extension of the DICE model. Uncertainty analysis conveys to decision makers the uncertainty in model predictions, avoiding the risk of overconfidence in model forecasts. However, for a full understanding and corroboration of model results, analysts might be willing to (and ought to) identify the model inputs that influence the model results the most (key drivers), the direction of change associated with the variation of a given input and the overall model structure (interaction analysis).

We are aware of only three studies devoted to the application of methods similar to the ones proposed in this paper to study the effects of uncertainty on IAMs. van Vuuren et al. (2008) apply a probabilistic approach to an energy model, Hof et al. (2008) use the FAIR IAM and Anthoff and Tol (2011) explicitly address the effects of uncertainty on the social cost of carbon (current damages caused by each unit of emissions) using the FUND model. In all cases, Monte Carlo simulations are used to propagate uncertainty and the results of those simulations are post-processed using either raw correlations or standardized regression coefficients to signal the magnitude of the impact that parameter uncertainty has on model outputs. The SA literature clearly describes the weaknesses of using correlations or standardized regression coefficients as a methodology for post-processing the Monte Carlo results. These limitations are mainly linked to their poor performance in the presence of non-linearities and interactions (Campolongo and Saltelli, 1997) so that several authors have argued in favor of the utilization of more robust methods (Sobol', 1993; Rabitz and Alis, 1999; Saltelli et al., 2004, 2008).

Generally speaking, IAMs are used to evaluate the long term implications of climate-economy interactions and to calculate the damages associated with current and future emissions and what an optimal policy intervention (usually in the form of a tax) might be. We scanned the existing literature of studies that calculate the social cost of carbon in order to understand how modelers use SA methods when making these calculations. Our starting point is the list of studies covered by Tol (2008) in his meta-analysis of the range of estimates of the social cost of carbon. Three IAMs emerge as the most widely applied and commonly cited in the literature: Richard Tol's FUND, Chris Hope's PAGE and the DICE model of William Nordhaus. In general, the studies based on the FUND model handle uncertainty through the use of probabilistic distributions for certain model inputs. They generally employ some type of scenario analysis where results from different modeling runs are displayed using alternative values of model inputs of interest, such as discount rates. The PAGE model uses probabilistic distributions of inputs to account for uncertainty and presents results as probability distributions rather than single deterministic values. As to the DICE model, Nordhaus (1994) performs a SA in two steps. A global OFAT SA is first performed to determine the eight most influential inputs. These are then subjected to uncertainty analysis, with distributions assigned based on expert opinion. Uncertainty analysis is also performed in later studies of the model (Nordhaus, 2008). In general, many of the cited studies in Tol (2008) acknowledge the existence of uncertainty and attempt to perform some type of SA. This is usually accomplished by altering the values of a certain targeted inputs, often the discount rate and/or climate sensitivity, to test outcomes under different scenarios. The tendency is, therefore, to perform specific sensitivity questions, and not to let the model undergo a systematic investigation through SA methods.

The results of an overview of over 1000 papers published in the literature in prominent environmental and climate change journals is presented in a literature review performed by the authors and briefly presented in the appendix. The findings are in line with those of Saltelli and Annoni (2010) who show that most of the papers use OFAT methods, whose limitations are well known, while no application of global methods is recorded. Leaving the details to the appendix, the main findings of the search can be summarized as follows:

1. there is no unique way of understanding the term sensitivity analysis;
2. there is no common protocol for the use of SA methods. Thus, one does not have a shared set of tools to answer questions such as: what is the exact sensitivity analysis question that we are asking? what is the most appropriate method for the task?
3. the use of sensitivity methods differs across environmental and climate change journals. Global SA methods are attracting growing interest in the environmental modeling field. The recent appearance of these techniques in multiple papers published in a special issue on integrated modeling and policy issues in *Environmental Modeling & Software* (Volume 26, Issue 12) is a clear sign of the fact that this topic is hot in the environmental modeling community. However, in

journals that focus on climate change modeling (and especially for IAMs), while SA is often evoked or performed - and in some instances extensive investigations are carried out - the application of the most recently developed (and most informative) methods is still lacking.

4. There is an issue of awareness: sensitivity methods suitable to answer questions that climate change modelers ask are already available, but they appear not to be known to analysts.

3 GLOBAL SENSITIVITY ANALYSIS: SETTINGS AND METHODS

By global sensitivity analysis one means the probabilistic evaluation of a model sensitivity, in the presence of uncertainty in the model inputs. Formally, let $\Omega_X \subseteq R^n$ be the set of possible values that the model inputs can assume and $(\Omega_X, B(\Omega_X), P_X)$ denote the corresponding probability space. $F_{\mathbf{X}}(x)$ denotes the joint cumulative distribution function (CDF) of the model inputs and $f_{\mathbf{X}}(x)$ their density. $F_{\mathbf{X}}(x)$ is assigned by the analyst based on her state-of-knowledge about the model inputs. **x** denotes one of the possible realizations of the random vector **X**. We denote by:

$$y = g(\mathbf{x}) : \Omega_X \subseteq \mathbf{R}^n \rightarrow \mathbf{R} \tag{1}$$

the relationship that links the model inputs to the model output. The analytic expression of g is, usually, not explicitly known, being the result of elaborate calculations of complex computer codes. Because **X** is uncertain, *y* becomes a random variable, denoted by *Y*. The associated probability space is (Ω_Y, B_Y, P_Y), $F_Y(y)$ and $f_Y(y)$ denote the CDF and density of *Y*, respectively.

Performing a global SA means propagating uncertainty through the model, either analytically or numerically, to obtain *y* (Reilly et al., 2001; Forest et al., 2002; Bernstein et al., 2009; Webster, 2009). Numerical uncertainty propagation goes under the heading of Monte Carlo simulation, which covers the various sampling generation methods (Sobol' quasi random sequences, Latin Hypercube sampling, etc.). Independently of the random number generation algorithm, a sample of size *N* is produced and the model is evaluated *N* times. The cost of the analysis is *C=N* model runs, as noted in the next section.

An integral part of a global SA is the statement of the goals of the analysis in order to identify the most appropriate methods and avoid misleading conclusions. In this respect Saltelli and Tarantola (2002) introduce the concept of SA setting (see also Saltelli et al., 2004, 2008; Borgonovo and Tarantola, 2008; Borgonovo, 2010). A setting is a way to frame the SA quest so as to clearly identify its objectives. In this paper we make use of the following settings:

1. Model structure: to determine whether the endogenous variable behavior is the result of the superimposition of individual effects or it is driven by interactions;
2. Direction of change: to determine what is the expected direction of change in the endogenous variable due to individual or simultaneous changes in the exogenous model inputs;
3. Factor Prioritization: to determine the key uncertainty drivers, namely the factors on which resources should be focused in data and information collection to most effectively reduce variability in a model's predictions.

We now discuss each of these settings in turn.

3.1 MODEL STRUCTURE

The understanding of the structure of a model input-output mapping requires the assessment of interactions, as shown below. Assume that the model mapping $g(\mathbf{x})$ is integrable (thus, in principle even non-smooth). Then, $g(\mathbf{x})$ can be written exactly as (Efron and Stein, 1981); Sobol', 1993; Rabitz and Alis, 1999):

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{i<j}^{n} g_{i,j}(x_i, x_j) + \ldots + g_{1,2,\ldots,n}(x_1, x_2, \ldots, x_n) \tag{2}$$

where:

$$\begin{cases} g_0 = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x})] = \int \ldots \int g(\mathbf{x}) \prod_{i=1}^{n} dF_i \\ g_i(x_i) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i] - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s\neq i}^{n} dF_s \\ g_{i,j}(x_i, x_j) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i, X_j = x_j] - g_i(x_i) - g_j(x_j) - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s\neq i,j}^{n} dF_s \\ \ldots\ldots \end{cases} \tag{3}$$

Eq. (2) is called the high-dimensional model representation (HDMR) of $g(\mathbf{x})$ (Rabitz and Alis, 1999). In (3) g_0 is the average value of *y* over Ω_Y; $g_i(x_i)$ accounts for the individual effect of X_i, $g_{i,j}(x_i, x_j)$ accounts for the residual interactions of model inputs X_i, X_j, and so on. Eq. (2) states that $g(\mathbf{x})$ is exactly reconstructed by the sum of the functions in the right hand side of (3). Eqs. (2) and (3) provide the multivariate "integral" expansion of $g(\mathbf{x})$.

Assume now that $g(\mathbf{x})$ is square integrable. Then, by the orthogonality of the functions in eq. (3), by subtracting g_0 from $g(\mathbf{x})$ one obtains the complete decomposition of the variance of Y:

$$\mathbb{V}_{\mathbf{X}}[Y] = \sum_{i=1}^{n} V_i + \sum_{i<j}^{n} V_{ij} + \ldots + V_{1,2,\ldots,n} \tag{4}$$

where the generic term of order *r* in eq. (4) is given by:

$$V_{i_1,i_2,\ldots,i_r} = \int g^2_{i_1,i_2,\ldots,i_r} \mathrm{d}F_{i_1} \mathrm{d}F_{i_2} \ldots \mathrm{d}F_{i_r} \tag{5}$$

On the basis of (4) and (5), Sobol' (1993) introduced the sensitivity indices of order *r* defined as:

$$S_{i_1,\, i_2,\ldots i_r} \equiv \frac{V_{i_1,\, i_2,\ldots i_r}}{\mathbb{V}_{\boldsymbol{X}}[Y]} \tag{6}$$

Special attention is deserved by the first and the total order sensitivity indices, defined respectively as:

$$S_l^1 \equiv \frac{V_l}{\mathbb{V}_{\boldsymbol{X}}[Y]} = \frac{\mathbb{V}_{X_l}[\mathbb{E}\{Y \mid X_l\}]}{\mathbb{V}_{\boldsymbol{X}}[Y]} \qquad (l = 1, \ldots, n) \tag{7}$$

and:

$$S_l^1 \equiv \sum_{r=2}^{n} \sum_{l,i_2,\ldots,i_r}^{n} \frac{V_{l,i_2,\ldots,i_r}}{\mathbb{V}_{\boldsymbol{X}}[Y]} = \frac{\mathbb{E}_{\mathbf{X}_{\sim l}}[\mathbb{V}_{X_l}\{Y \mid X_l\}]}{\mathbb{V}_{\boldsymbol{X}}[Y]} \qquad (l = 1, \ldots, n) \tag{8}$$

In eq. (8), the symbol $\mathbf{X}_{\sim l}$ denotes all factors but X_l. Disentangling the contribution of single variables and of interactions to the overall model variability (Setting 1) is quite naturally addressed by applying the functional ANOVA decomposition and the associated sensitivity measures reported in (6), (7) and (8). The sum of these variance-based sensitivity measures provides indications on model structure. In the case $\sum_{i=1}^{n} S_i^1 = 1$ the model is additive, that is, its response is the exact superimposition of the individual effects of the exogenous variable. Conversely, if $\sum_{i=1}^{n} S_i^1 < 1$ interaction effects are present. The lower the sum of the first order indices is, the higher the relevance of interactions.

3.2 DIRECTION OF CHANGE

Setting 2, the expected direction of change in the endogenous variable, can be addressed through the investigation of functions $g_0 + g_i(x_i)$. Note that, from the second equation in (3), we have:

$$\mathbb{E}[g(\mathbf{x}) \mid X_i = x_i] = g_i(x_i) + g_0 \tag{9}$$

Thus, $g_0 + g_i(x_i)$ represents the conditional expectation of $g(\mathbf{x})$ as a function of x_i. In particular, if $g(\mathbf{x})$ is additive, then $g_i(x_i) + g_0$ displayes the the exact dependence of Y on X_i. Thus, we are

able to understand whether Y is a monotonic function of X_i with no approximation and for all values of X_i. If $g(\mathbf{x})$ is not additive, then eq. (9) is a trend line that allows us to understand the dependence of Y on X_i as all possible values of the remaining model inputs are averaged. Thus, there is a difference between comparative statics in the sense of Samuelson (1947) and comparative statics performed using an integral approach, like the one adopted here. By differential comparative statics, one obtains a local information, namely the variation rate of *Y* around one given point in the input parameter space for a small variation in X_i. On the basis of eq. (9) one obtains a global information about what happens to $g(\mathbf{x})$ as X_i varies over its entire range.

3.3 FACTOR PRIORITIZATION

The identification of key uncertainty drivers (setting 3) may appear to be linked to the discussion above on variance decomposition, suggesting that a critical parameter could be the one which has a significant impact on the endogenous variable(s) variance. However, it is well known that variance is not a good summary measure of uncertainty, especially when the distributions are skewed or multimodal, and when inputs are correlated, which is likely the to be the case in many natural phenomena, including climate change. In this case, Borgonovo (2007) proposes a better suited sensitivity measure, defined as follows:

$$\delta_l = \frac{1}{2}\mathbb{E}_l[s_l(x_l)] \tag{10}$$

where:

$$s_l(x_l) = \left| f_Y(y) - f_{Y|X_l=x_l}(y) \right| \tag{11}$$

$s_i(x_i)$ measures the separation between the unconditional distribution of the model output [$f_Y(y)$] and the conditional model output distribution given that model input X_i is fixed at x_i [$f_{Y|X_i=x_i}(y)$]. Geometrically, $s_i(x_i)$ is the area enclosed between $f_Y(y)$ and $f_{Y|X_i=x_i}(y)$.

It can be shown that δ possesses the following convenient properties: (i) normalization to unity, i.e. $0 \le \delta_i \le 1, \quad i = 1, 2, ..., n$; (ii) joint normalization: $\delta_{1,2,...,n} = 1$; (iii) scale invariance: if $u(Y)$ and $t(Y)$ are two monotonic functions, then $\delta_i^{u(Y)} = \delta_i^{t(Y)} = \delta_i^Y$. The first property states that each exogenous model input has an ”importance index”', which lies between 0 and 1. In particular, an exogenous model input X_i has null importance if Y and X_i are independent. The second property states that the joint importance of all model inputs is unity. The third property of scale invariance is desirable for two aspects. The first one emerges in numerical estimation. In several applications the

output of a Monte Carlo simulation is sparse or spans a wide range. This could bring about inaccurate estimation of sensitivity measures. To improve numerical precision, analysts often resort to a transformation of the model output (usually, a log-transformation). Scale invariance insures that, after any monotonic transformation the results of SA remain unaltered. The second reason is that in many applications the model output is valued through a utility function. It is a well-known principle of economic thery that utility functions have an ordinal, not cardinal, meaning, so that they can be freely modified through monotonic transformations. Scale invariance, then, insures that results of the sensitivity analysis remain valid for any chosen monotonic utility function. For further discussion on the decision-making implications of this result see Baucells and Borgonovo (2012).

4 ESTIMATION

The estimation of the sensitivity measures proposed above is analytically feasible only in very few instances and with simple mathematical expressions that usually do not represent an environmental or economic problem. For IAMS, which are complex simulation tools encoded in dedicated software, the estimation is forcedly numerical.

An algorithm that strictly reproduces the definitions in eqs (7), (8) and (10) - brute force estimation - is associated with a computational cost equal to:

$$C = Nn^2 \tag{12}$$

model runs, where N is the sample size of Monte Carlo simulation and n the number of factors. N should be chosen in such a way as to ensure estimation accuracy. At N=1000 C is greater than one million model runs, making the estimation prohibitive for any IAM.

However, computation reduction results have been reached in the global SA literature. They have led to a drastic reduction in the estimation of variance-based indices, lowering C to:

$$C = N(n+2) \tag{13}$$

model runs for estimating all first and total order sensitivity measures (Saltelli, 2002; Saltelli et al., 2010; Campolongo et al., 2011).

The sampling plans in Castaings et al. (2012) lower the computational cost of the δ-importance measure to:

$$C = N \cdot r \tag{14}$$

where r is the number of replicates.

Note that an analyst pursuing these estimation strategies has possibly to run two different sets of numerical experiments, one to estimate S_l^1 and S_l^T and one to estimate δ_l . Moreover, in both cases, the sampling plans would differ from the utilization of a simple Monte Carlo uncertainty propagation.

In this paper we pursue an alternative strategy which enables us to obtain all sensitivity measures from the same dataset and at the lowest possible computational cost. Recent work has produced notable advances in this respect, lowering the computational cost to:

$$C = N \tag{15}$$

There are two main ways to proceed. The first foresees making use of a meta-model.[1] Here we make use of the GUI-HDMR software of Ziehn and Tomlin (2009). The software allows the estimation of Sobol' sensitivity measures of orders 1 and 2 from the component functions $g_i(x_i)$, $g_{i,j}(x_i, x)$ which are obtained by fitting orthonormal bases, through a system of equations of the type:

$$\begin{aligned} g_i(x_i) &\approx \textstyle\sum_{r=1}^{h} \alpha_r^i \phi_r(x_i) \\ g_{i,j}(x_i, x_j) &\approx \textstyle\sum_{p=1}^{h'} \sum_{q=1}^{h''} \beta_{p,q}^{i,j} \phi_p(x_i) \phi_q(x_i) \end{aligned} \tag{16}$$

where $\phi_r(x_i)$ is an element of a family of orthonormal polynomials, α_r^i , $\beta_{p,q}^{i,j}$ are the corresponding coefficients, h, h' and h'' determine the order of the expansion (see for further details Ziehn and Tomlin, 2009).

Following the Cut-HDMR approach (Rabitz and Alıs, 1999) one then obtains insights on model structure, through knowledge of the variance-based sensitivity indices, and on monotonicity, by plotting the $g_i(x_i)$ functions.

The second way is to utilize orthogonal projections and is used in Plischke et al. (2012). This technique allows one to estimate variance-based sensitivity measures and δ_l . The method consists of a reordering of the data set to form a scatterplot $X_i \oplus y$, followed by a partitioning of the data set. The method works as a post-processing algorithm and the estimation is direct, without the need of a meta-model. We shall make use of both the Cut-HDMR meta-model and Plischke et al's method in our analysis. The advantage of combining the proposed approaches is that one retrieves all the discussed insights without having to utilize an ad-hoc sampling scheme and using the dataset produced by Monte Carlo simulation. Thus, we add to current practice where Monte Carlo propagation has become part of the standard way of operating. In the next section, we discuss the application of the proposed approach to the DICE model.

[1] We recall Kriging (see Kleijnen, 2009), Gaussian emulation (Oakely and O'Hagan, 2004), Cut-HDMR (Rabitz and Alis, 1999; Ziehn and Tomlin, 2010), polynomial chaos expansion (Sudret, 2008), and state-dependent parameter modelling (Ratto and Pagano, 2010).

5 GLOBAL SENSITIVITY ANALYSIS OF THE DICE MODEL

To illustrate the proposed methodology we have chosen the DICE model to perform a global SA. DICE is one of the most widely acknowledged IAMs due to the expertise of William Nordhaus, "whose careful pragmatic modeling throughout his DICE series of IAMs has long set a standard..." (Weitzman, 2007, p.713). Nordhaus (2008) characterizes the DICE model as "a global model that aggregates different countries into a single level of output, capital stock, technology, and emissions. The estimates for the global aggregates are built up from data that include all major countries, and the specification allows for differentiated responses and technological growth." (p.33) DICE has been evolving since the early 1990s with many refinements and adaptations to answer specific research questions. We use Version 2007.delta.8 of DICE.[2] We do not go into details concerning the nature and general structure of the model because it is comprehensively described in Nordhaus (2008). We limit ourselves to note that the input-output mapping is composed of a series of interconnected equations (or submodels), thus generating a multiplicity of outputs. These depend on the model inputs, which are reported in Table A1 of the Appendix.

The presentation of the results of our global SA exercise in divided in two parts. The first set of results stems from a comparison of our methodology with the sensitivity of the DICE model directly performed by Nordhaus (2008), where only certain inputs were subjected to uncertainty and sensitivity analysis. The second set presents results for the dataset obtained when uncertainty in all inputs is considered and the outputs considered are inter-generational welfare (utility), social cost of carbon in 2005, global atmospheric temperature in 2105, global emission level in 2105, and the optimal carbon tax for 2015. Among the many outputs produced by DICE we focus on the ones just mentioned because they are relevant for policy purposes and grant comparison with previous SA performed using the same model. From an operating viewpoint we proceed in two steps. Firstly we propagate uncertainty in the model inputs through DICE via Monte Carlo simulation; secondly, we post-process the corresponding dataset using the methods described in the previous sections.

5.1 A COMPARATIVE ANALYSIS

Our reference point is the sensitivity analysis of the DICE model performed in Chapter 7 of Nordhaus (2008). It relies on a pre-screening exercise performed in Nordhaus (1994) and identifies 8 inputs as key uncertainty drivers, which should be subjected to increased scrutiny. We take the outcome of the pre-screening exercise for granted and use the same probability distributions for the 8 inputs as in Nordhaus (2008) in order to offer a comparison of the insights that can be obtained by applying the methods discussed in this paper. We will remove the restriction on the number of factors later on in

[2] Version 2007.delta.8 can be downloaded from Nordhaus' website http://nordhaus.econ.yale.edu/DICE2007_short.gms.

the section. Results obtained when all model inputs are varied are then compared to results obtained when the subset of the a priori selected factors is considered.

Table 1 displays the results of an OFAT analysis of the DICE model originally presented in Tables 7-2 and 7-3 of Nordhaus (2008). It conveys the impact that the value of a given model input has on a model output as the input moves from one to six standard deviations from the assumed mean value. Let $\mathbf{x^0}$ denote the mean value of the model inputs and $(x_i^0 + k\sigma_i, \mathbf{x}_{\square i}^0)$ the point obtained by moving only X_i by k standard deviations (k=1, 2, 3, 4, 5, 6).[3] The percentage changes from $g(\mathbf{x}^0)$, $\Delta_i^k = \frac{g(x_i^0 + k\sigma_i, \mathbf{x}_{\square i}^0) - g(\mathbf{x}^0)}{g(\mathbf{x}^0)}$, are taken by Nordhaus (2008) sensitivity measures and displayed both in absolute and relative terms. We are then in an OFAT framework.

The numerical values in the Table 1 display the value of the social cost of carbon in 2005 (top panel) and of global emissions in 2105 (bottom panel) when the value of the parameters is altered. For example, when the model inputs are at their mean value, the social cost of carbon in 2005 is $g(\mathbf{x^0})$=\$28.10. When the value of GA0, the growth in total factor productivity, is altered by one standard deviation the value of social cost of carbon increases to \$36.07, a 28% increase from the mean value. The table shows that for the social cost of carbon in 2005 the quadratic coefficient in the damage function (A2) has the largest effect as k (i.e., the distance from the mean value) varies. The global population limit (POPSYM) and parameter in the carbon cycle (b12) have a less relevant effect, while uncertainty in the price of the carbon free backstop technology (PBACK), the rate of decarbonization in the economy (GSIGMA) and the total amount of fossil fuels available for consumption (FOSSLIM) have no effect at all on the social cost of carbon in 2005. When the output of interest is global emissions in 2105, the growth of total factor productivity (GA0) has undoubtedly the largest influence, while the rate of decarbonization (GSIGMA) and asymptotic population (POPASYM) have smaller effects. Note that the influence of a model input depends on the output of concern.

[3] The results are shown only when the parameters move away from the mean value in the positive direction (k=[1,6]) rather than in both ones (k=[-3,3]) since, "the results are sufficiently linear that this displays the patterns accurately" (Nordhaus, 2008, p.129).

Table 1: Summary of Nordhaus (2008)' SA Results

SOCIAL COST OF CARBON 2005								
Standard Deviation	GA0	GSIGMA	T2XCO2	A2	PBACK	POPASYM	b12	FOSSLIM
0	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)
1	36.07 (28)	28.27 (1)	38.07 (35)	40.99 (35)	28.1 (0)	32.14 (14)	29.16 (4)	28.1 (0)
2	48.08 (71)	28.43 (1)	46.44 (65)	53.89 (65)	28.1 (0)	35.91 (28)	30.32 (8)	28.1 (0)
3	51.21 (82)	28.6 (2)	53.49 (90)	66.8 (90)	28.1 (0)	39.44 (40)	31.61 (12)	28.1 (0)
4	54.68 (95)	28.76 (2)	59.47 (112)	79.73 (112)	28.1 (0)	42.75 (52)	33.04 (18)	28.1 (0)
5	58.52 (108)	28.92 (3)	64.59 (130)	92.66 (130)	28.1 (0)	45.84 (63)	34.62 (23)	28.1 (0)
6	62.8 (123)	29.09 (4)	69.03 (146)	105.61 (146)	28.11 (0)	48.75 (73)	36.39 (30)	28.1 (0)
GLOBAL EMISSIONS 2105								
0	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)
1	30.99 (62)	21.95 (15)	19.18 (1)	19.18 (1)	19.08 (0)	22.84 (20)	19.08 (0)	19.08 (0)
2	50.19 (163)	25.19 (32)	19.28 (1)	19.28 (1)	19.08 (0)	26.42 (38)	19.09 (0)	19.08 (0)
3	78.2 (310)	28.83 (51)	19.38 (2)	19.38 (2)	19.08 (0)	29.84 (56)	19.1 (0)	19.08 (0)
4	103.92 (445)	32.91 (72)	19.48 (2)	19.48 (2)	19.08 (0)	33.06 (73)	19.1 (0)	19.08 (0)
5	65.19 (242)	37.36 (96)	19.59 (3)	19.59 (3)	19.07 (0)	36.08 (89)	19.1 (0)	19.08 (0)
6	24.61 (29)	42.22 (121)	19.7 (3)	19.7 (3)	19.07 (0)	38.9 (104)	19.11 (0)	19.08 (0)

Source: Nordhaus (2008) and our own calculations.

It is apparent that this type of OFAT approach does not grant a robust identification of the key uncertainty drivers due to the instability of the implied sensitivity rankings with respect to the variation range. For the social cost of carbon in 2005 the coefficient in the damage function (A2) is always the most influential input, regardless of the distance from the mean value. Conversely, at one standard deviation (k =1) climate sensitivity (T2XCO2) is ranked third behind the growth rate of total factor productivity (GA0) in the magnitude of the change from the mean value, but the ranking is reversed at two standard deviations from the mean value (k=2) and then again as k gets larger. The reason is that the method is intrinsically local and does not account for the entire variability of the uncertain inputs. This limitation is overcome by the use of a global method. By propagating uncertainty one obtains the distribution of the outputs of interest, as in Figure 7-2 of Nordhaus (2008). Then, our approach allows one to obtain the factor ranking by post-processing this data set. Using the algorithm of Plischke et al. (2012) we obtain robust information about the key uncertainty drivers. Figure 1 reports the estimates of Borgonovo (2007)'s δ_l (see eq. (10) for the pre-selected model inputs again with respect to the social cost of carbon and global emissions. At the top of each bar the 90% confidence intervals obtained from 500 bootstrap replicates using the bias-reducing estimator proposed in Plischke et al. (2012) are displayed.

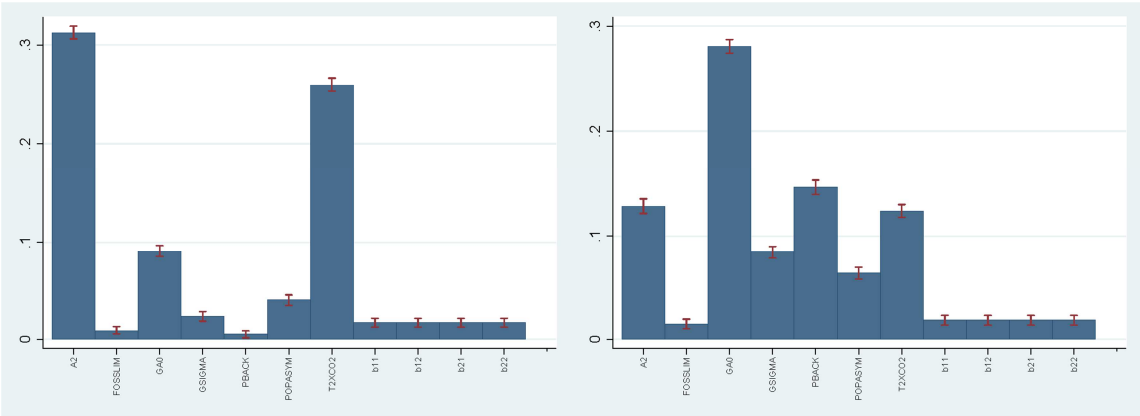

Figure 1: δ_l 's with Pre-Selected Model Inputs for the Social Cost of Carbon (left) and Global Emissions (right).

The results displayed in Figure 1 show that at N=10000 model runs the global sensitivity measures are estimated with high confidence. The ranking is consistent with Table 1 for the social cost of carbon, insofar the coefficient in the damage function (A2), climate sensitivity (T2XCO2) and the growth rate of total factor productivity (GA0) are the most influential inputs. However, the message is not the same with respect to the level of global emissions in 2105. Table 1 provided the message that uncertainty in the growth rate of total

factor productivity was most important. We see from the figure that while δ_l for this model input is still the greatest by magnitude, uncertainty in other factors is not insignificant in influencing future emissions.

We observe a further difference by post-processing the outcomes of the Monte Carlo simulations. The results in Figure 1 do not pre-suppose an analysis in which the factor is set at pre-determined values proportional to its standard deviation. They instead consider the entire range of uncertainty in both the input and output factors. They also display, via the bootstrap, the confidence level we have in the obtained ranking. There is no analogous information in the OFAT analysis of Nordhaus (2008).

OFAT methods do not reveal interactions, but these can also be extracted from the same dataset produced by the uncertainty analysis. To that end we apply the GUI-HDMR Matlab code of Ziehn and Tomlin (2009b). By the analysis of the data set generated from Monte Carlo simulations we obtain values of the second order sensitivity indices of $\sum_{i,j=1}^{n} S_{i,j} = 0.4233$, when the output is the social cost of carbon in 2005 and of $\sum_{i,j=1}^{n} S_{i,j} = 0.6053$ when the output is global emissions in 2105. These values indicate that the model responds non-additively to the inputs and that interaction effects are relevant. The interaction between the growth in the rate of factor productivity (GA0) and the price of the backstop technology (PBACK) is the most influential on the social cost of carbon in 2005, while the interaction between the coefficient in the damage function (A2) and the climate sensitivity parameter (T2XCO2) have the strongest effect on the level of global emissions in 2105. Figure 2 displays the HDMR of the most influential interactions for the social cost of carbon in 2005.

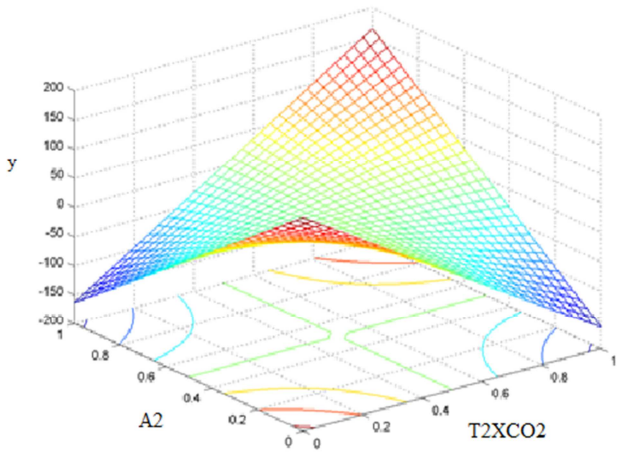

Figure 2: Input Interactions for the Social Cost of Carbon

The figure shows the plot of the bivariate function $g_{A2,T2XC02}(A2, T2XC02)$ representing the interactions between A2 and T2XCO2, when the output is the social cost of carbon. This second order function is neither

convex nor concave and non-monotone. Also, note that the functions are not strictly positive or negative across the entire uncertainty ranges for the interacting inputs. When both inputs are at the upper end of their uncertainty ranges, the interactive effect is a negative one, while at the lower end of the ranges the interactive effect has the opposite sign. As a result the second order effects can have either an amplifying or dampening effect on the first order individual effects. The application of these methods thus provides a quantitative dimension to Nordhaus (2008)' statement that "an examination of all the uncertain model inputs taken together ... may produce unexpected results because of the interactions among the model inputs and the non-linearity in the DICE model" (p.134).

By plotting the first order terms in the HDMR decomposition one gathers insights about the direction of change and monotonicity when factors vary individually. Figure 3 shows the impact of variation in climate sensitivity (T2XCO2) on the social cost of carbon (left panel) and of the initial growth rate of the technology (GA0) on the level of global emissions (right panel), the two model inputs with most significant HDMR effects.

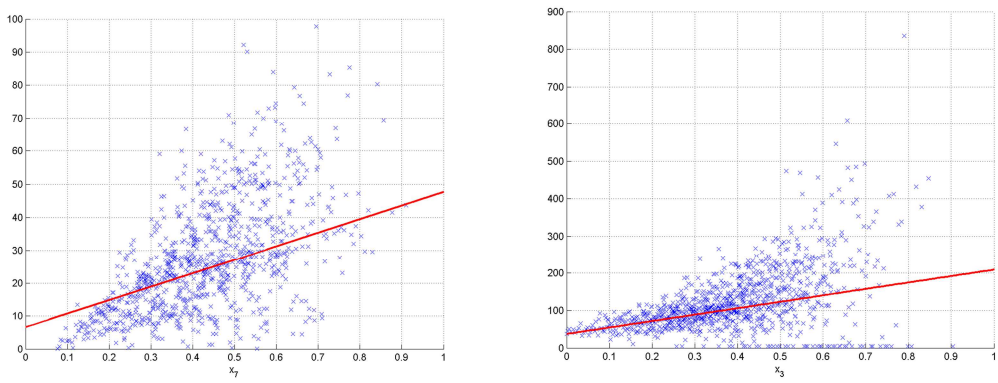

Figure 3: Impact of $g_{T\,2XCO2}(T\,2XCO2)$ on Social Cost of Carbon (left) and $g_{GA0}(GA0)$ on Global Emissions (right).

As expected, as the value of the climate sensitivity parameter rises so does the social cost of carbon, and the same relationship holds for the effect of the rate of technological growth on total emissions at the end of the century. For both model outcomes the first order functions $g_i(x_i)$ are monotonic. In particular, one can determine whether they are increasing or decreasing for all factors (this information is not reported here for brevity). However, by looking at these graphs a decision-maker can gain insights about whether a factor tends to increase the social cost of carbon (or global emissions) on average.

The above considerations show a major advantage of the methods proposed and described in this paper, in that significant interactions can be identified explicitly, rather than simply acknowledged or speculated upon, and the direction of the interactive effect can be observed. In both cases, the interactive effects are non-linear

and additive to overall variation in the outcome variables. This simple comparative exercise is an indicator of the potential advantages of using global SA methods over OFAT methods.

In the next section we expand the analysis considering all model inputs. There is no additional computational burden by subjecting all model inputs to uncertainty analysis, as the DICE model solves the same number of times in the Monte Carlo simulations regardless of the number of model inputs that are altered. The major advantage, of course, is that any biases or seemingly justifiable reasons to limit the number of model inputs subjected to uncertainty analysis might impaire the analyst's view of how the model itself acts under uncertainty.

5.2 RESULTS WHEN UNCERTAINTY IN ALL MODEL INPUTS IS CONSIDERED

We now drop the restriction on the number of inputs and allow all DICE model inputs to vary. For demonstration purposes, we assign them the range of ± 10% the original value, using a uniform distribution. Choosing the width of the interval is admittedly arbitrary, so that we repeat the analysis using intervals of 5% and 20% with consistent results that are not reported here but are available upon request. In the reminder of the analysis we will only display the results identifying the key uncertainty drivers. The HDMR images when all model inputs are varied are similar to those in Figures 2 and 3 where first order effects are monotonic and second order interaction effects are significant and non-monotonic.[4]

We post-process the data from the Monte Carlo simulations using the methods of Plischke et al. (2012) and calculate the bootstrapped confidence intervals for δ_i. Figure **Errore. L'origine riferimento non è stata trovata.** displays the importance of the model inputs when the output of interest is inter-generational utility.

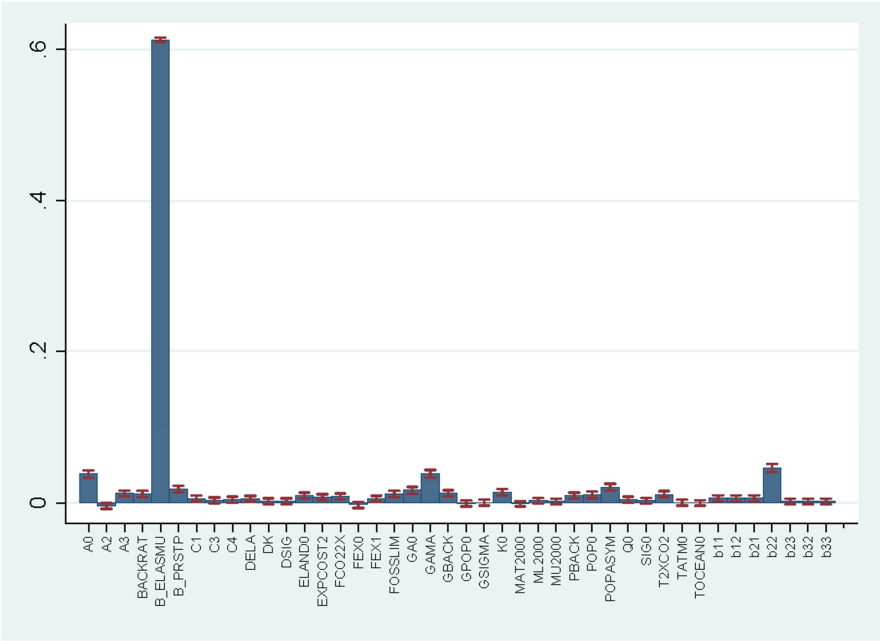

Figure 4: δ_i 's with Intergenerational Utility as Model Output

[4] The images are available from the authors upon request.

The dominant input factor driving variation in the model output is quite clearly the elasticity of the marginal utility of consumption (B_ELASMU). We note that B_ELASMU appears in the DICE model as the variable μ in the Ramsey's equation:

$$r = \rho + \mu \cdot b \tag{17}$$

where r is the social discount rate, ρ is the rate of pure time preference, b is the growth rate of consumption per capita and μ is the elasticity of marginal utility of consumption. μ is also known as the coefficient of relative risk aversion, because the DICE model uses a constant elasticity of substitution (CES) utility function where $L(t)$ denotes labor force or population:

$$U\left[c(t), L(t)\right] = L(t)\left[c(t)^{1-\mu} / (1-\mu)\right] \tag{18}$$

B_ELASMU (μ) determines the shape of the utility function and the relationship between consumption increases and utility or welfare.

Sterner and Persson (2008) succinctly explain the economic logic behind assumptions related to values for μ: "the higher the value of μ, the less we care for a dollar more of consumption as we become richer. Since we expect that we will be richer in the future, when climate damages will be felt, a higher μ also implies that damages will be valued lower. Thus, a higher value of μ implies less greenhouse gas abatement today, unless for some reason we will be poorer rather than richer in the future. In this case, a higher μ would give higher damage values, which would justify more abatement" (p.66).

The findings displayed in Figure 4 provide key support to the argument that decisions affecting the components of the Ramsey equation and implicitly the discount rate are of primary importance in IAM exercises. However, a philosophical discussion on these aspects is beyond the scope of this paper and interested readers have plenty of well-conceived studies to consult on this topic. Nordhaus (2008) uses model inputs for the Ramsey equation that sum up to coincide with observed market rates of return on capital. However, Newell and Pizer (2003) show that market rates of return are not stable over longer periods of time, and the effects of minor changes in the relevant model inputs can have significant effects on model outcomes, as documented in Figure 4. Our analysis confirms that this model input is key for results but also allows to understand how important this parameter is. It is also useful to understand how this factor interacts with other factors in the model. The strongest interaction is with the exponent in the Cobb-Douglas production function (GAMA) and the image of the HDMR is similar to that of Figure 2 where at certain combinations of the model inputs in their uncertainty ranges the interactive effect can be either positive or negative.

Economists and policy makers are not only interested in drivers of inter-generational utility, which in itself is an abstract concept, but they focus also on pragmatic, relevant calculations that are of concern - such as the level of total emissions at the end of the century – and actionable – such as magnitudes of carbon taxes. We

thus now turn to the results of a SA performed along the lines just followed, with the major difference that we now subject all inputs to uncertainty propagation and not only those pre-selected by Nordhaus (1994). For ease of exposition we focus on the social cost of carbon in 2005, global emissions at the end of the century, and the rise of global atmospheric temperature at the end of the century relative to 1900, as done in Nordhaus (2008). We will also consider the effects of uncertainty on the optimal carbon tax in 2015, since that should be of concern to policy makers in the near term. Table 2 contains descriptive information about how uncertainty affects these outcomes. The ranges of outcomes are non-negligible, as a world that is 3.5 degrees warmer is likely very different than one that is 2 degrees warmer. They are also in line with the IPCC best estimates of what is required if excessive negative consequences from climate change are to be avoided. The same can be said for the range of damages current emissions are inflicting, global emissions in 2105 and the appropriate carbon tax in 2015 that would put us on an optimal trajectory, as calculated by the DICE model.

Table 2: Results from Monte Carlo Simulations

Variable	Observations	Mean	Standard Deviation	Min	Max
Global Average Temperature Rise by 2105	10,000	2.708	0.215	2.014	3.542
Social Cost of Carbon in 2005	10,000	27.252	6.126	13.130	55.170
Global Emissions in 2015	10,000	120.664	18.580	75.083	207.545
Carbon Tax in 2015	10,000	40.044	9.113	18.159	81.988

The table conveys information similar to that presented in Figure 7-2 in Nordhaus (2008) or to the finding of Arigoni Ortiz et al. (2011), who vary specific model inputs probabilistically in a SA performed on an adaptation of DICE. By going one step further and post processing the results of the probabilistic uncertainty analysis much insight is gained. Figure 5 displays the point estimates for δ_l and the bootstrapped confidence interval when the outcome of interest is the social cost of carbon in 2005 (left panel) and global emissions in 2105 (right panel).

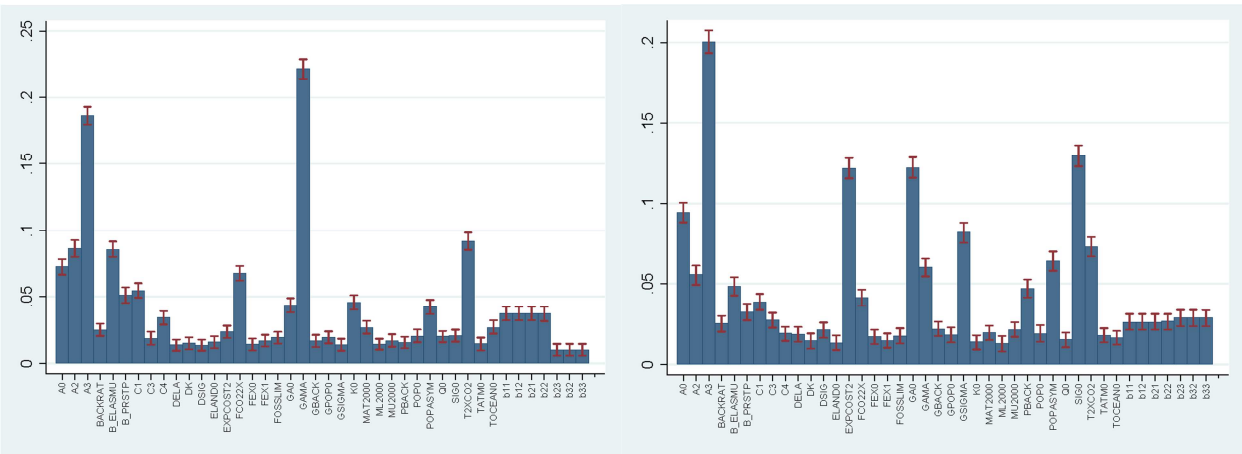

Figure 5: δ_l 's with Social Cost of Carbon (left panel) and Global Emissions (right panel) as Model Outputs

Uncertainty in the elasticity of capital in the production function (GAMA) has the strongest influence on the social cost of carbon in 2005, followed by the exponent in the damage function (A3). The climate sensitivity parameter (T2XCO2), the elasticity of marginal utility (B_ELASMU) and the coefficient in the damage function (A2) are all roughly equally, but much less influential. The same cannot be said for the model inputs that influence the level of global emissions at the end of the century. Of primary importance is the exponent in the damage function (A3), followed by the emissions intensity of the economy in 2005 (SIG0), the initial growth rate of technological progress per decade (GA0) and the exponent in the cost control function (EXPCOST2).

The left panel of Figure 6 shows that uncertainty in the initial levels of total factor productivity (A0) influence atmospheric temperature in 2105 most, followed closely by climate sensitivity (T2XCO2), emissions intensity of the economy in 2005 (SIG0), capital elasticity in the production function (GAMA) and the exponent in the cost control function (EXPCOST2). The same cannot be said of the effects of uncertainty on the calculation of the optimal carbon tax level for 2015 where the exponent in the damage function (A3) is the most influential, followed by capital elasticity (GAMA), initial levels of total factor productivity (A0), elasticity of marginal utility of consumption (B_ELASMU), and the climate sensitivity parameter (T2XCO2).

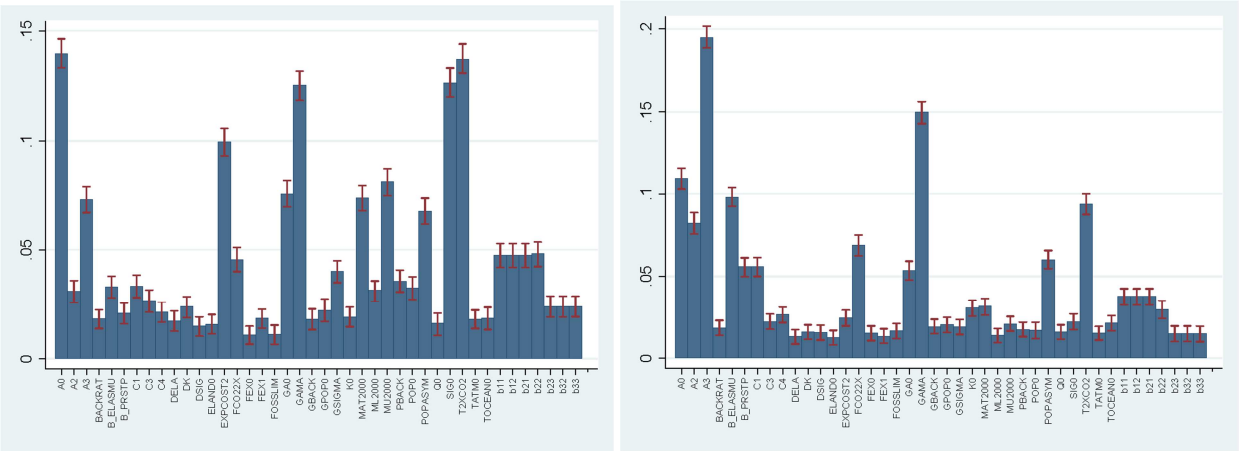

Figure 6: δ_l 's with Atmospheric Temperature (left panel) and
Optimal Carbon Tax (right panel) as Model Outputs

The SA results for these four policy relevant outcomes from the DICE model deliver two important messages. The first is that the relevant model inputs are identified quantitatively rather than through a pre-selection process. The messages here can be compared to the findings of Nordhaus and Popp (1997) where the goal was to find which model inputs should be investigated further in order to reduce uncertainty, according to the costs associated with uncertainty itself. The second point is highlighted by comparing the results from these analyses with the information presented in Table 3, where the eight model inputs examined in Nordhaus (2008) are presented with the ranking of their δ_l 's.

Table 3: Nordhaus (2008)' Pre-Selected Parameters SA Rank

Pre-Selected Variables	Social Cost of Carbon (2005)	Global Emissions (2105)	Atmospheric Temperature (2105)	Carbon Tax (2015)
A2	22	5	6	10
GA0	7	11	11	3
FOSSLIM	42	26	30	34
GSIGMA	16	36	26	6
PBACK	17	32	29	12
POPASYM	10	12	8	8
T2XCO2	2	3	5	7
B12	12	13	12	21

It is clear that the pre-selection of model inputs performed in Nordhaus (1994) was correct in including the climate sensitivity parameter (T2XCO2), the coefficient in the damage function (A2) and the initial growth rate in total factor productivity (GA0) since they each rank in the top ten factors for at least two of the policy relevant model outcomes of our approach. The remaining preselected inputs are not nearly as influential and for optimal uncertainty management more useful information could have been obtained if different inputs had been pre-selected and subjected to increased analysis, or given priority in information and data collection if that is a previously limiting factor.

As a next step we consider the rankings of all the delta scores for each uncertain model input when different outcomes are considered. The full list of rankings is provided in Table A2 of the Appendix. To conserve on space Table 4 summarizes this information reporting the rank correlations and Savage score correlations for each of the output. The rank correlations are computed considering the vector of the ranks of X_i with respect to each of the output. A correlation equal to unity implies that that for the two model outputs under consideration the most and least relevant factors are exactly the same. Each entry in Table 4 displays the raw correlations as first entry and the Savage score separated by a "|". Savage scores place emphasis on the agreement of the key (higher ranked) uncertainty drivers, while raw correlations indicate the relationship between all model inputs (Iman and Conover, 1987; Campolongo and Saltelli, 1997; Kleijnen and Helton, 1999). Let R_i be the rank of X_i. Then, the Savage Score of X_i is: $SS_i = \sum_{h=R_i}^{n} \frac{1}{h}$. For instance, a factor ranking first out of 51 has a Savage score of 4.52, a factor ranking second a score of 3.52, and so on.

When comparing rank correlations to Savage Score correlations we have two cases: a) if the rank correlation value is smaller than the corresponding Savage score correlation value, then there is higher agreement on the most important model inputs rather than across all inputs; b) the converse is true if the rank correlations are higher than the Savage score ones.

Table 4: Overall Correlations and Savage Scores for Model Outputs

	Atmospheric Temperature	Social Cost of Carbon (2005)	Carbon Tax (2015)	Global Emissions (2015)	Utility
Atmospheric Temperature	1				
Social Cost of Carbon (2005)	0.709|0.613	1			
Carbon Tax (2015)	0.789|0.669	0.958|0.951	1		
Global Emissions (2015)	0.816|0.718	0.657|0.599	0.774|0.717	1	
Utility	0.545|0.430	0.731|0.645	0.669|0.622	0.554|0.388	1

Overall Table 4 indicates that the important factors for utility are not the same as those affecting other policy relevant outcomes. Among the policy relevant outcomes, the social cost of carbon in 2005 and the optimal carbon tax share the strongest correlation of common drivers of variation, followed by global emissions and atmospheric temperature in 2105.

In general the model inputs can be split into the group of speculative parameters where the value is not empirically known and calculated through projections and the group of inputs that are econometric in nature and depend on statistical analysis. It is common to take the econometric group of model inputs as given and instead focus on the speculative inputs when performing sensitivity or scenario analysis. However, we have shown that uncertainty in both types of inputs proves to be influential in affecting model outcomes. Thus, global SA should be performed considering all model inputs, before deeming a set of inputs as influential, since assessing key drivers without an extensive quantitative analysis might lead to misleading conclusions.

6 CONCLUSIONS

This paper has demonstrated the usefulness of global sensitivity analysis methods in the area of integrated assessment modeling for climate change economics. It has shown that at the same computational cost of a standard uncertainty analysis one can obtain robust insights on direction of change, model structure (interactions) and key uncertainty drivers by applying recently developed methods. These insights provide analysts with a deeper understanding of a model's behavior and allow them to robustly identify the factors on which to focus additional data collection.

We have discussed both numerical and methodological aspects of the approach using DICE, one of the most popular models for climate change policy analysis. The results show that uncertainty in the elasticity of the marginal utility of consumption, which influences the discount rate applied, is by far the most influential parameter in affecting the dependent variable in the objective function of the model. The key uncertainty drivers have been also identified with respect to more pragmatic policy relevant model outputs. Differences in ranking of inputs with respect to the model outputs have been analyzed.

The results of this paper highlight the merits of performing global sensitivity analysis alongside other types of scenario analysis to explore different outcomes given different parameter values in the model. The most highly visible recent analysis of IAMs in the climate change literature revolved around what type of scenario

should be considered as a most reasonable informer for policy. The authors of the Stern Review claim that a scenario with low discount rates and strong inter-generational equity is the correct basis, while others avoid the ’normative’ discussions of discount rates, using instead observable market rates of return and arriving at much different conclusions and policy recommendations. While this highlights the usefulness of varied modeling strategies for different policy or scientific questions, our exercise has shown the benefits of using global sensitivity analysis methods since the two approaches are not interchangeable and important information can be taken from both. Lastly, global sensitivity analysis along the lines presented here could be fruitfully conducted on other classes of models routinely used in climate change policy analysis, from computable general equilibrium models for impact assessment to energy system techno-economic models. These are all topics for future research.

APPENDIX: A LITERATURE SEARCH ON THE USE OF METHODS FOR SENSITIVITY ANALYSIS IN CLIMATE CHANGE AND ENVIRONMENTAL MODELLING JOURNALS

A database of 1190 published articles was analyzed to investigate in how many cases sensitivity is intended as a structured approach to model output analysis and, in these cases, how many papers effectively use the most informative methods.[5] The following table summarizes the findings.

Table: Summary of a Survey on the Use of Sensitivity Analysis in Articles Published in Prominent Environmental and Climate Change Journals

Journal Title	Paper with Word "Sensitivity" in Title, Abstract, or Keywords	Papers Applying Global SA Methods	Year of Publication of Paper with Systematic SA	Type of Applied Approach
Climatic Change	230	0	-	-
Energy Economics	56	1	2007	ad hoc, local
Environmental Modeling and Assessment	40	6	2006,2007,2008,2009,2010	OFAT, ad hoc, non-parametric, variance-based
Environmental Modeling and Software	137	37	From 1999 to 2010	local, screening, non-parametric, variance-based
Environmental Research Letters	22	1	2008	variance-based
Global Environmental Change	3	1	2010	screening, variance-based
Journal of Hydraulic Engineering	87	4	1993, 1996, 2005, 2009	local, screening, non-parametric, variance-based
Resource and Energy Economics	17	-	-	-
The Energy Journal	20	-	-	-
Water Research	578	3	2009, 2010	non-parametric, variance-based
TOTAL	1190	53		

In *The Energy Journal* 20 articles have "sensitivity" in their title abstract or keywords. In none of them a systematic approach to sensitivity analysis in conjunction with uncertainty is performed.

In *Energy Economics* 56 papers have the term "sensitivity" in either title, abstract or keywords. Of them, only one utilizes a somewhat systematic sensitivity analysis, namely, Tarancon Moran and Del Rio Gonzalez (2007) where an ad-hoc approach to sensitivity performed for a specific problem is used. However, the

[5] In the table the journals *Science* and *Nature* are not listed because a review of the use of sensitivity methods in these journals is offered elsewhere (Saltelli and Annoni, 2010). The list of journals considered here does not pretend to be exhaustive.

approach does not consider uncertainty. The remaining 55 either consider sensitivity as a synonym of dependence or investigate sensitivity to specific assumptions. No works make use of recently developed global sensitivity methods for uncertainty management.

In *Resource and Energy Economics* the picture is similar, with 17 papers having the word sensitivity in their title, abstracts or keywords, but with none of them actually using a systematic approach.

In *Environmental Research Letters* 22 studies have the word sensitivity in either title, abstract or keywords. Among these works, Lobell and Burke (2008) ask the question of the relative importance of uncertainties in temperature and precipitation measurements in leading to uncertainties in the agricultural impacts of climate change. The authors develop an analytic equation and study its variance. The approach is, in principle, very similar to a variance-based global sensitivity and they obtain a particular case of expression (4) in the text of the paper, where interactions are neglected.

In *Environmental Modeling and Assessment*, 40 papers have the word sensitivity in ether title, abstract or keywords. Of these 6 operate a systematic sensitivity analysis utilizing nonparametric methods (Fullerton et al., 2010), variance-based methods (Avagliano and Parrella, 2009), OFAT methods (Hess et al., 2008), screening methods (Braddock and Schreider, 2006; Matthews et al., 2007), and a model-specific method in Webby et al. (2009).

In *Water Research* 578 papers have the word "sensitivity" in their title abstract and keywords, spanning a time period from 1967 to 2010. Of these, three contributions use a joint sensitivity and uncertainty analysis approach. Two works (Sourisseau et al., 2008; Sin et al., 2011) make use of non-parametric techniques, one work makes use of variance based techniques (Neumann et al., 2009). The remaining papers either apply sensitivity to answer ad-hoc questions - thus, no systematic insight or integration to uncertainty is performed - or the word sensitivity is used as a synonym of dependence.

In the *Journal of Hydraulic Engineering* 87 papers have the word sensitivity in their title. Several of them apply either local methods or OFAT methods. Four papers utilize a systematic approach to sensitivity in the presence of uncertainty. Variance based methods are used in Hall et al. (2005) and in Hall et al. (2009). Indelman et al. (1996) and Yeh and Tung (1993) apply both variance-based and non-parametric methods.

In *Environmental Modeling & Software* the number of publications devoted to a systematic use of uncertainty and sensitivity analysis has been recently growing, actually boosting in 2010, as we are to see. Out of 137 papers containing sensitivity analysis in either title, abstract and keywords and spanning the years 1998-2010, 37 make use of systematic sensitivity analysis methods. Of these, 10 were published or appeared in year 2010, 3 in 2009, 4 in 2008, 3 in 2007 and the remainder in the previous years. For space reasons we cannot list them all here, but we can offer some examples. Papers using screening methods are for instance Campolongo and Braddock (1999) where the Morris method is applied in the SA of the IMAGE model for greenhouse emissions and Cryer and Havens (1999) where a fractional factorial scheme based on Plackett and Burman (PB) design is used in the study of an air pollution model. They are also applied in Saltelli et al. (2010) and Saltelli and Annoni (2010). Non-parametric methods are applied in Manache and Melching (2008) in the context of the uncertainty and global SA of water quality models. Examples of papers applying

variance-based methods are Varella et al. (2010), Confalonieri et al. (2010) in the global SA of crop models, Estrada and Diaz (2010) in the global SA of an eutrophication model. We also recall the algebraic approach proposed by Norton (2008) for general environmental models. We also note the recently published special issue (Volume 26, issue 12) on integrated modeling and policy issues that highlighted the use of global sensitivity analysis for environmental models.

A similar search in *Climatic Change* reveals 230 papers with the word sensitivity as part of their title or abstract or keywords. Sensitivity appears in the expression "climate change sensitivity", where it is used as a synonym for dependence. It is also used in the context of "feedback and climate sensitivity", as in Schwartz (2010). 6 articles have the locution "sensitivity analysis" in their title: these are O'Neill and Wexler (2000), Van Der Linden et al. (2003), Gerlagh and van der Zwaan (2004), Keller et al. (2005), Zickfeld and Bruckner (2008), and Bormann (2011). Bormann (2011) actually compares different models in their sensitivity to climate change, with the word sensitivity used as a synonym for dependence. In Van Der Linden et al. (2003) sensitivity is performed by using an OFAT approach. In Keller et al. (2005) the sensitivity of a snow cover in mean, minimum and maximum temperature alone and a change in mean temperature combined with a precipitation change of +10% in winter and -10% in summer is investigated. Comprehensive sensitivity studies are performed in O'Neill and Wexler (2000), Gerlagh and Van Der Zwaan (2004) and Zickfeld and Bruckner (2008), where series of combinations of uncertain variables with changes in inputs and scenarios are explored. However, the most recently developed methods for sensitivity analysis in conjunction with uncertainty have not been applied yet.

Finally, in the journal *Global Environmental Change*, only 3 papers have the word sensitivity in their title and only one of them, Saltelli and D'Hombres (2010), proposes the use of sensitivity analysis methods. Indeed, those authors offer a strong critique of the way in which sensitivity analysis has been used in the debate about climate change in the works of Stern and Taylor (2007) and Nordhaus (2007a). Saltelli and D'Hombres (2010) come to the conclusion that the way in which sensitivity analysis was used by those authors proved ineffective in supporting their conclusions.

Table A1: DICE Model Inputs

SA Number	Parameter	Description
1	A0	Initial level of total factor productivity
2	A1	Damage intercept
3	A2	Damage quadratic term
4	A3	Damage exponent
5	BACKRAT	Ratio initial to final backstop cost
6	ELASMU	Elasticity of marginal utility of consumption
7	B_PRSTP	Initial rate of social time preference per year
8	C1	Climate-equation coefficient for upper level
9	C3	Transfer coefficient upper to lower stratum
10	C4	Transfer coeffic for lower level
11	DELA	Decline rate of technological change per decade
12	DK	Depreciation rate on capital per year
13	DPARTFRACT	Decline rate of participation
14	DSIG	Decline rate of decarbonization per decade
15	DSIG2	Quadratic term in decarbonization
16	ELAND0	Carbon emissions from land 2005(GtC per decade)
17	EXPCOST2	Exponent of control cost function
18	FCO22X	Estimated forcings of equilibrium co2 doubling
19	FEX0	Estimate of 2000 forcings of non-CO2 GHG
20	FEX1	Estimate of 2100 forcings of non-CO2 GHG
21	FOSSLIM	Maximum cumulative extraction fossil fuels
22	GA0	Initial growth rate for technology per decade
23	GAMA	Capital elasticity in production function
24	GBACK	Initial cost decline backstop pc per decade
25	GPOP0	Growth rate of population per decade
26	GSIGMA	Initial growth of sigma per decade
27	K0	2005 value capital trill 2005 US dollars
28	LIMMIU	Upper limit on control rate
29	MAT2000	Concentration in atmosphere 2005 (GtC)
30	ML2000	Concentration in lower strata 2005 (GtC)
31	MU2000	Concentration in upper strata 2005 (GtC)
32	PARTFRACT1	Fraction of emissions under control regime 2005
33	PARTFRACT2	Fraction of emissions under control regime 2015
34	PARTFRACT21	Fraction of emissions under control regime 2205
35	PBACK	Cost of backstop 2005 per tC 2005
36	POP0	2005 world population millions
37	POPASYM	Asymptotic population
38	Q0	2005 world gross output trillion 2005 US dollars
39	SIG0	CO2-equivalent emissions-GNP ratio 2005
40	T2XCO2	Equilibrium temperature impact of CO2 doubling C
41	TATM0	2000 atmospheric temperature change (C) from 1900
42	TOCEAN0	2000 lower stratospheric temperature change (C) from 1900
43	b11	Carbon cycle transition matrix
44	b12	Carbon cycle transition matrix
45	b21	Carbon cycle transition matrix
46	b22	Carbon cycle transition matrix
47	b23	Carbon cycle transition matrix
48	b32	Carbon cycle transition matrix
49	b33	Carbon cycle transition matrix
50	scale1	Scaling coefficient in the objective function
51	scale2	Scaling coefficient in the objective function

Table A2: All Delta Rankings for Each DICE Model Input

Model Input	Atmospheric Temperature	Social Cost of Carbon 2005	Carbon Tax	Global Emissions	Utility
A0	1	6	3	5	4
A1	43	43	43	43	43
A2	22	5	6	10	42
A3	8	2	1	1	10
BACKRAT	34	20	27	24	12
ELASMU	19	4	4	11	1
B_PRSTP	29	9	10	15	6
C1	18	8	9	14	23
C3	23	27	20	19	27
C4	30	17	18	28	25
DELA	37	39	40	31	22
DK	24	33	33	38	31
DPARTFRACT	43	43	43	43	43
DSIG	40	37	31	26	32
DSIG2	43	43	43	43	43
ELAND0	39	31	42	42	15
EXPCOST2	5	21	19	4	21
FCO22X	15	7	7	13	17
FEX0	41	38	35	35	41
FEX1	33	30	41	39	24
FOSSLIM	42	26	30	34	13
GA0	7	11	11	3	7
GAMA	4	1	2	9	3
GBACK	36	29	25	25	9
GPOP0	28	25	24	32	39
GSIGMA	16	36	26	6	38
K0	31	10	16	40	8
LIMMIU	43	43	43	43	43
MAT2000	9	19	15	29	40
ML2000	21	35	39	41	30
MU2000	6	28	23	27	29
PARTFRACT1	43	43	43	43	43
PARTFRACT2	43	43	43	43	43
PARTFRACT21	43	43	43	43	43
PBACK	17	32	29	12	16
POP0	20	23	28	30	11
POPASYM	10	12	8	8	5
Q0	38	24	32	36	26
SIG0	3	22	21	2	28
T2XCO2	2	3	5	7	14
TATM0	35	34	34	33	36
TOCEAN0	32	18	22	37	37
b11	12	13	12	21	18
b12	12	13	12	21	18
b21	12	13	12	21	18
b22	11	16	17	20	2
b23	26	40	36	16	34
b32	26	40	36	16	34
b33	25	42	38	18	33
scale1	43	43	43	43	43
scale2	43	43	43	43	43

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