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Im-perfectly competitive contract markets for electricity*

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Abstract

Notwithstanding academic and regulatory interests as well as empirical evidence, to date the effect of contracts on competition in electricity markets is a very controversial issue. We suggest an original approach to shed light on this debate. Modeling competition by mean of conjectural variations we demonstrate that anti-competitive effects follow the upsurge of discrimination practices. Altering the time distribution of the perfect arbitrage constraint (i.e. *ex-post* in spite of *ex-ante*) we put a bridge between IO and financial models on price manipulation. Endogenizing forward demand we provide a rationale for shifting dominant attitudes to forward markets. Finally, studying sequential choices we balance quantity and price competition with Stackelberg leadership. Simulations and qualitative estimates support our findings.

Keywords: Electricity Markets, Imperfect Competition, Forward and Futures

JEL Codes: L13, L44, L94

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1 Introduction

Do contracts enhance competition in electricity markets? Answers to this question are very controversial: economic and political debates on this issue seem both characterized by a paradox.

Regarding the theoretical point of view, at the beginnings of the 90's, Allaz and Vila (1992, 1993, hereafter, AV) have shown the appealingness of long-term commitments as market power mitigation devices. Generators offering contracts can behave more aggressively on the correlated output market because they need to refund buyers for high strike prices. Furthermore they expect pre-commitments would increase their market share, as the rival reacts by adjusting its production accordingly. However a *prisoner's dilemma* type of effect is underway. Unilateral pre-commitments give short operators a Stackelberg advantage, but since everybody can do so at equilibrium, no one is expected to succeed in acquiring an effective leadership. Consequently competition is enhanced and the output price declines. After roughly a decade of stagnation, academics have recently renewed the interest in forward trading. Much of this work has focused on the electricity industry, as it features the AV modelling framework (oligopoly suppliers, homogenous commodity, robust forward markets). While some experimental and econometric works (see Wolak 2002, Brandts et al, 2006, Le Coq 2005) have succeed in providing empirical evidence to AV's predictions, several theoretical models have questioned the foundation of the setting.

Regulatory and antitrust authorities have followed an opposite evolution. The 90's were characterized by a generalized policy mistrust in forward trading. The opposition entailed the following motives: contracts slow down entry, decrease the transparency and reduce the liquidity of output markets. However the recent experience in liberalizing strategic sectors such as gas and electricity, has contributed to altering the past attitude of policy makers (Bushnell, *forthcoming*). To date, competition in energy markets is far from being achieved and forward contracts are being considered *ex-ante*, by the regulatory authorities, as substitutes to structural divestiture *ex-post*. This has been the case, for instance, in Belgium, France and Denmark, where Virtual Power plants (VPP) have obliged the incumbent generator to sell part of its production capacity to other market participants as an alternative to capacity divestiture.[1] At the end of 2006, for instance, VPP have been designed in Italy by the Antitrust Authority as a valuable behavioural measure to counteract the abuse of dominant position by ENEL.[2]

The examination of the most recent empirical evidence provided by markets for electricity could serve in order to enrich the theoretical and the competition policy debates. The creation of spot or wholesale markets to trade electricity on a day ahead basis has represented one of the most important milestones of the sector liberalization. All the forms of contracting, physical and financial, standardized or over-the counter (OTC), parallel spot markets and have steadily developed.[3] As we will argument later in the paper (see Section 3), the DG Competition Sector Inquiry (2007) points out that electricity companies do not just sell their surplus generation or cover their supply commitments but engage in arbitrage deals. In fact, most speculative trading, to the extent that it exists, tends to involve long positions (that is buying contracts) in the forward market which will further

[1] Virtual Power plants are (bundles of) call options. They are promises (or obligations) to deliver a specific amount of electrical energy at a time in the future whenever and up to the extent the buyer finds it optimal to do so. Therefore in contrast to forward and futures contracts, Virtual Power plants allow the buyer to decide when it is reasonable to exercise the option and up to which extent (e.g. buyers are not obliged to use their VPPs always at full capacity). As a reward for such flexibility, whenever the buyer exercises the option he has to refund the seller a production cost equals the strike price of the contract. For more details on VPP, see Bonacina et al. (2006).

[2] VPP designed by the Antitrust Authority are two-way differential contracts that dispose an obligation on ENEL to give up the extra-profits eventually earned on the Power Exchange, due to its ability to raise prices in that market, but at the same time give him the right to receive a financial compensation when the market price is lower than the strike price . The market price of the underlying electricity is referred to the South area if Italy, which is the one in which ENEL was found to be pivotal for more than 95% of the hours.

[3] A forward and/or futures contract is a promise (and obligation) to deliver a specific amount of a commodity and/or of an asset at a time in the future. Forward contracts contain detailed specifications about the underlying commodity and/or asset and the delivery process (e.g. the grade/quality of the traded good, the delivery date and the delivery location). By converse futures contracts have a standard form and some allowance of flexibility. The standardization facilitates the trade of futures on organized exchanges. Therefore, according to the benefits and costs of specificity versus flexibility, agents choose between forward and futures contracts if both the mechanisms are available. It is usually recognized that agents engage in forward contracting when they expect the physical trade of the commodity at the delivery date. In fact futures positions are often offset before the contract expires. OTC transactions do not per se involve organised marketplaces. Rules governing the trade are typically derived from practice and based on industry agreements.

increase forward prices. Spot and contract marketplaces differ in terms of the number of participants and liquidity. Prices for day-ahead base load delivery observed on the power exchanges and contract markets are very closely correlated both in terms of development and levels, some differences being due to the existence of risk premia. Concentration in national wholesale markets (whether in terms of ownership of generation assets or in terms of trade in a given product) gives scope for exercising market power. Oligopoly competition in spot markets features Cournot models when generators are capacity constrained, and Bertrand settings when the constraint is relaxed.[4] Furthermore, market participants have raised some concerns about concentration forward markets. In principle, generators with market power on spot markets have ample opportunity to also exercise their influence on forward prices, but evidence is hard to find.

It is not the purpose of this paper to downplay the progress made in the liberalization exercise, but to analyze the impact of contracts on quantity and price competition in the electricity market, by including the most relevant characteristics of trading in spot and forward markets, at the light of the most recent empirical evidence. Consistently with the AV, we consider a duopolistic industry producing a homogeneous commodity which may be traded on either a long-term or a spot markets which are assumed to open sequentially. To allow firms to take long positions, contracts in our analysis encompass both forward (or physical) contracts and futures (financial and standardized contracts). We model competition in the spot market by means of conjectural variations, to conduct robustness investigations and to estimate the likely impact of alternative concentration levels on equilibria. Conjectural variations are also used to model competition in the forward market, featuring the possibility of an imperfectly competitive forward market put forth by the DG Competition sector inquiry.[5]

As a preliminary result, we show that long-term commitments are neither absolutely for nor exclusively against competition *per se.* The two aspects are sides of the same coin. In static settings where the forward/spot interaction occurs only once upon the whole time schedule, the output market structure (e.g. the degree and the mode of competition and the technological structure of the industry) exerts a major impact on the sub-game perfect

[4] This is consistent with several theoretical analyses on electricity spot markets (among others, see Crampes and Creti, 2005, and Fabra *et al.*, 2006).

[5] Consistently with empirical evidence in power markets we exclude monopolistic positions in both forward and spot markets.

equilibrium. The main result of Section (4) is to show that price competition is a necessary and sufficient condition to separate spot and forward decisions. This is due to the perfect substitutability of the commodity delivered in the two markets which verifies when agents are risk-neutral, rational and forward looking. Moreover, fiercer competition in the spot market leads to reduction in aggregate output whenever duopolists are allowed to take long positions, that is to act as net buyers of the commodity to withhold production in the spot market. By causing an upsurge of capacity withholding strategies, long positions subvert the dynamic and exacerbate market power even in perfectly competitive markets. This is the main determinant of the anti-competitive effect of forward pre-commitments. Our analysis therefore discloses the likely motives for the contradictions that have emerged in both the economic and the policy debates.

Our original contribution to the literature on contracts and market power is to show that forward market can be used to sustain duopolistic positions in the spot market, as generators, recognizing that demand in the forward or futures markets is not simply the residual from sales in the spot markets, but a proper function of forward price, can exploit profit opportunities by strategically altering the time distribution of the perfect arbitrage constraint, i.e. *ex-post* in spite of *ex-ante* (Section 5). In the absence of speculators that can exploit arbitrage opportunities between forward and spot market prices, generators could exercise their market power by imposing intertemporal price discrimination. This possibility is limited, however, by the fact that at equilibrium, absent uncertainty and risk aversion, spot and forward market price must converge, otherwise contracts do not have any economic reason. We solve a constrained profit maximization problem by imposing the equality between spot and forward prices and prove that there is a range of parameter values (demand and costs) for which trading in forward market increases spot profits. In particular, firms can optimally choose long positions to create spot demand and thus sustain market price. Whenever buying contracts is not possible (for instance, with physical commitments), firms ensure positive contract coverage, still optimally restricting the share of the signed agreement with respect to AV benchmark. When the arbitrage condition holds *ex-ante*, instead, the forward market will absorb any contract quantity that does not show up in the spot market and this eliminates any profitable deviation from AV setting.

The mode of competition is crucial. Consider Bertrand setting. Since output and therefore spot market prices are independent of forward market equilibria, when the perfect arbitrage condition binds *ex-ante*, perfectly competitive - Bertrand-like - duopolists are neutral to any amount of pre-committed contracts. Results do not hold in contexts where the same constraint is assumed to bind *ex-post*. In fact, we find out that a given amount of forward is delivered at equilibrium. The rationale is in the potential for arbitrage profits which increase the appealingness of signing contracts. With Cournot competition in the spot market, differences in the operation of the perfect arbitrage constraint change the responsiveness of the setting to forward market equilibria. The *ex-post* actual gives the positive value of the Lagrangian multipliers associated to the convergence between spot and forward prices such that, when the constraint is binding, the second stage best reply becomes a "market clearing condition" (Kamat and Oren, 2004) between the price obtained at the second stage of the model (spot market competition) and the forward demand function. Interestingly, this constraint becomes the relevant reaction function. Thus the multiplier associated to perfect arbitrage in *ex-post* settings is the shadow value of the foregone profit in the contract market, if intertemporal differentiation was allowed. Since the multiplier represents a fraction of contracts that a firm would sign to influence forward market price, its sign is endogenous and depends on the value of demand and the cost parameters.

A further important result is that at equilibrium with spot and forward price arbitraged, both in *ex-ante* and in *ex-post* settings, market power in forward markets is not sustainable. In other words, the degree of competition in such markets does not influence contracting. Only in *ex-post* setting, when we allow for price differentials between spot and forward markets (that is with a Lagrangian multiplier equal to zero), contracts do depend on the degree of market power. However, we know that absent risk aversion and uncertainty, such as price differences cannot be sustained at equilibrium. Therefore, our result supports the EC inquiry, in that almost no trading platform has been identified where operators systematically have a dominant position on supply or demand.

To better understand how market power influence forward trading, we have also studied a Stackelberg model (Section 6), a modelling framework that has been neglected so far by the traditional IO literature. Despite the leadership in the spot game, forward contracts go on serving as a binding device. Short producers behave more aggressively in the output market not only for the refunding obligation but also because of the awaited increase in the market share. However, when there is an *ex-post* arbitrage condition, we find that, similarly to the simultaneous case, the equilibrium outcomes can change, altering the profit distribution among the leader and the follower and increasing the spot price.

The paper is organized as follows. Related literature is summarized in Section (2), whereas Section (3) presents relevant findings on spot and contract markets analyzed by the DG Competition at the European level. Section (4) shows the results form the literature by introducing conjectural variations to model competition in the spot market. Section (5) studies the impact of imperfect competition in the contract markets, in simultaneous games, whereas Section (6) focuses on sequential settings. Section (7) briefly concludes. Proofs are relegated to Appendix A. In Appendix B, we compute and simulate the solution of the Cournot model under specific functional forms and to provide detailed comparisons to AV model. Similar calculations are provided for the Stackelberg model.

2 Related literature

To date, different models criticize the main assumptions of AV, whose conclusions are very sensitive to the modelling choices.[6] The most obvious objection comes from altering the mode of competition, the issue to which our analysis is devoted.

Mahenc and Salanié (2004, hereafter, MS) have demonstrated that when the spot game is characterized by price competition, the opportunity of committing forward can reduce competition and create upward pressures on prices. Selling contracts is a strictly dominated strategy. This finding does not sound as counterintuitive and the rationale may be summarized as follows. Price setting attitudes (e.g. Bertrand competition) are socially efficient, with the prevailing market equilibria equal to the first best; therefore market separation via long-term contracts becomes a mean for escaping the privately unsuitable competitive outcome for the most profitable Cournot counterpart. The effect is amplified by the complementarity in players' actions, as a price increase from one firm entails a subsequent increase by the rival. In other words when implemented in Bertrand settings, forward trading engenders dominance in correlated markets. Therefore, in equilibrium competition is relaxed and profits increase. As explained in the introduction, by using conjectural variations, we reconcile AV and MS models, by showing that in fact they

[6] The pro-competitive effect of contracts is weakened by non-observability of firms strategies (Hughes and Kao, 1997) and insufficient demand of contracts due to risk-averse retailers (Green, 2004). Considering transmission constraints also limits the efficiency of contracts (Kamat and Oren, 2004). Modifying structural conditions, that is endogenizing installed capacity (Adilov, 2005) or considering entry (Newbery 1997), Gans et al.1998), also contribute to weaken AV's predictions. For an extensive survey of these models, see Bonacina et al. (2006).

explain two aspects of the same mechanism. Therefore, the contradiction between the results obtained, under Cournot and under Bertrand competition is respectively more apparent, than real.[7]

To our knowledge, two papers consider the specific characteristics of forward demand. Powell (1993) shows that, under demand uncertainty and risk-aversion on the buyer side, the efficiency-enhancing effect of a contract market is realized only if the upstream market is perfectly competitive, i.e. generators are Bertrand players. Green (2004) extends the analysis of the demand-side strategic choice of contracts and finds that uncertainty, risk-aversion and retail competition adversely affect agents' incentives to enter into long-term agreements. Entry pressure limits the ability of risk-averse incumbent retailers to pass on the costs of forward contracts. In conjunction with uncertainty in spot demand, such a threat is expected to lead to partial (instead of full) cover of retailing sales. Consequently, when there is a competitive retail market firms enter the spot market with fewer forward sales. This in turn reduces the aggressiveness of producers and therefore the prisoner's dilemma effect that emerged in AV. Also notice that due to retailers' risk attitudes, Green (2004) finds that at equilibrium the forward price falls below the expected spot price and that contracts are negatively affected by the conjectural variation parameter modelling competition in the forward market. The results we obtain in *ex-post* settings are qualitatively similar to those of Powell and Green, in that profitable deviation from AV realizes with a lower contract coverage. However, we have weaker hypotheses as we do not need uncertainty in spot demand nor risk aversion: a demand for contracts negatively related to forward price suffices. Finally, as we eliminate price differentials between spot and forward prices, we rule out the possibility of varying the contract coverage with the degree of forward market concentration at the constrained equilibrium. For instance, in the Appendix, when using linear functional forms, we show that when perfect arbitrage is not

[7] There is one alternative branch of research that has questioned the effectiveness of contracting on competition. When linear supply function competition on the output market combines with Cournot behaviours in the forward stage as in Green (1999), it results in oligopolists not participating in the contract market (unless they can earn a hedging premium from selling to risk-averse buyers). In other words the system is neutral to pre-commitments, which in turn are ineffective as means for mitigating dominance. However it is our opinion that as long as to obtain analytical solutions the supply function approach merges with linear functional forms, we are incapable of clarifying which of the two hypotheses is the crucial determinant of the forward neutrality. For this reason, we will not further investigate supply function models.

binding, contracts are a decreasing function of the conjectural variation parameter in the forward market, as obtained by Green.

Finally, by considering the possibility of speculative trading by firms, we propose a bridge between the traditional IO literature and the financial literature on market manipulation, and in particular to those models, closer to the industrial economics perspective, where the key factor is the so called "corner and squeeze" strategy.[8] By cornering the futures market, firms artificially increase demand for the good. At the same time, firms, which have market power in the spot market, would restrict supply to squeeze the market. The combination of the two can cause substantial increases in price. In other words, dominant firms can amplify the price effects of withholding supply by becoming a large trader and simultaneously increasing demand. As a result, the possibility of having imperfectly competitive contract markets alters the dynamic response of the competitors in the spot market, and changes the equilibrium outcomes. Pirrong (1995, 2001) shows that a trader who can buy or sell an arbitrarily large number of futures contracts is able to influence the price at spot liquidation by demanding or selling too many units of the commodity in the delivery market. He can profit in equilibrium from the artificially high or low spot market price if he randomizes his order flow to hide behind the order flow of “noise traders” and if the supply curve in the delivery market is upward sloping. This results closely resembles to the solution of the simultaneous game with *ex-post* perfect arbitrage and long contracting positions by firms, with the noticeable difference that there is no uncertainty in our model.

This strand of the financial literature has also considered an environment where there is one dominant firm that can set the market price and a number of smaller firms that simply react to the price. This is similar to the framework we use in the Stackelberg model. Newbery (1984) and Anderson and Sundaraesen (1984) show that a dominant firm, by using derivatives, can achieve an outcome similar to the one of “predatory pricing”, lowering prices below marginal costs to force smaller firms out of the market. Under certain circumstances it would be rational for the dominant firm to lower futures prices,

[8] Financial models investigate three forms of manipulation: action-based, wherein actual or perceived asset values are changed; information-based, which involves rumor-mongering or expedient timing of information release; and trade-based, which depends upon buying or selling volumes. Research in finance has shown that markets can be manipulated if some agents have private information about prices: agents with inside information can profitably exploit their advantage by hiding behind the order flow of uninformed “noise traders” (see Hart (1977), Kyle (1985), Jarrow (1992, 1993), Allen and Gale (1992), Kumar and Seppi (1992)).

because the smaller firms' response to the lower prices would be to reduce output. As a result of this strategy, the market share of the dominant firm would increase. In our model, assuming that the leader is cost inefficient as it is generally the case in electricity markets, we do not leave room to predatory pricing strategies. However, as in Newbery (1984) and Anderson and Sundarasen (1984), in our work contracts alter the profits' distribution between the leader and the follower, but this can be in favour of the "small" firm. In particular, when the perfect arbitrage condition holds *ex-ante*, only the follower contracts to effectively counterbalance spot market dominance. In *ex-post* settings both firms engage in contracting and, depending on demand and cost parameters, they can increase profits.

3 Electricity contract markets in Europe

The DG Competition Sector Inquiry (2007) is one of the most thorough investigations in the Commission's history. Findings in the power sector of the preliminary report range from discontent at what is suspected as market abuse stemming from the dominant position of large generators on the power markets, to the linked problem of vertical integration, and difficulty in obtaining access to networks. Concerning in particular electricity contracts, the inquiry provides a thorough description of the main characteristics and the functioning of spot and forward markets.[9]

Although forward trading has developed differently across various countries, traded volumes have significantly raised during the last two years. In countries where market concentration is high, as for instance France and Belgium, trade remains quite low. Regarding the size of the spot and contract markets, it has been pointed out that the number of active participants on the power exchanges trading futures products is significantly lower than on the respective OTC markets. Nord Pool, together with the German OTC forward market, has the highest number of participants and also attracts the largest number of

[9]The inquiry specifies that "typical spot products on continental European markets are single hours or groups of hours, whereas forward products include weekly, monthly, quarterly and yearly products. Forward electricity can either be traded as a 'base' or a 'peak' contract. The term 'base' implies a continuous delivery throughout the delivery period (e.g. a month), whereas 'peak' typically only involves a delivery on business days from 08:00 till 20:00. The definitions and contract specifications may differ between countries. Electricity for spot and forward delivery can be traded on both power exchanges and over-the counter markets. Standardised forward contracts traded on exchanges are called futures. Contract specifications of exchange traded and OTC products are in practice very similar or identical allowing for efficient arbitrage".

financial traders, followed by the UK, France, Netherlands and Belgium. Moreover, on most spot and forward markets the vast majority of participants act in general as both sellers and buyers of electricity. The DG competition analysis shows that larger electricity companies do not just sell their surplus generation or cover their supply commitments but engage in arbitrage deals or take speculative positions. On the other hand smaller companies tend to be active on the wholesale market only to optimize their physical portfolios. Over the last years, traded spot volumes on exchanges are larger than brokered spot markets in most of the Member States examined. Thus market results on power exchanges seem to be setting the pace for the overall traded spot market. Finally, it is important to note that on most power exchanges a relatively small number of market participants' accounts for a large part of the overall spot volume traded on both the selling and buying side. This is especially true for OMEL in Spain, GME in Italy and Denmark West in the Nord Pool.

On the impact of contracts on competition, the EC inquiry reports that forward markets tend to take their cue from the spot market. Overall, the introduction of the forward market does not seem to create downward pressure on prices. Instead, forward and spot prices seem to converge. For example, with reference to the German market, the Sector Enquiry concludes that "prices on exchanges and forward markets go hand in hand".[10] This supports one of the main hypotheses that theoretical industrial organization models assume, that is perfect arbitrage between short-term and medium term prices.

In the influence it can have over prices in the OTC and power exchange markets, both through the abstaining of capacity and through the size of bid or offer it can make, large generators have the upper hand. A large number of participants and a healthy liquidity on the markets would serve to reduce the effects of any such play. However this result has not been obtained. Even in the spot market where the number of participants is much larger, a relatively small number of market participants' accounts for a large part of the overall spot volume traded on both the selling and buying side, according to the Commission.

The EC report rounded up views received on the functioning of spot and forward markets as such:

[10] In Bonacina et al. (2006) we have extensively investigated the French spot and contract markets. Using average monthly data from July 2003 to February 2007 and regressing the last settlement price of the monthly base load products on base load spot price, we find that, both for base load and peak load products, the medium-term contract is quite well explained by the short-term one.

"There is an oligopoly on the supply side (. . .) accounting for 80% of generation output."

"French and Belgian spot and forward markets are dominated by single players – thus distortions can easily occur there."

"Forward and futures prices in the German spot and forward market do not react to supply and demand. A very dry summer such as 2003 drives up prices, the end of the dry period should thus result in a price decrease. However a downward trend after a price peak is not observable. Obviously the few players at the power exchange are able to prevent price decreases by limiting the offer."

The EC concludes that generators with market power on spot markets have ample opportunity to also exercise their influence on forward prices. For example dominant operators could withhold a part of their generation capacity. This would not only raise spot prices but also change market participants' expectations of the development of this fundamental supply side factor resulting in higher forward prices. This challenges the AV predictions in that with perfect arbitrage, liquid forward markets do not seem to alleviate market power in the spot market. The objective of our analysis is therefore to explain such contradiction.

4 The analytical framework

The model rests upon the general assumptions presented hereafter. As noted in the introduction, functional specifications will be considered in later sections mainly to find out explicit solutions, clarify intuitions and/or derive appropriate policy recommendations. In the proceeding we detail the operation of the relevant markets (i.e. spot and forward markets) and the technological structure of the industry.

Consistently with AV, we consider a duopolistic industry producing a homogeneous commodity (i.e. energy) that may be traded on either a forward or a spot market which are assumed to open sequentially. The time schedule of the full game is presented in Fig.(1), the analytical formalization follows.

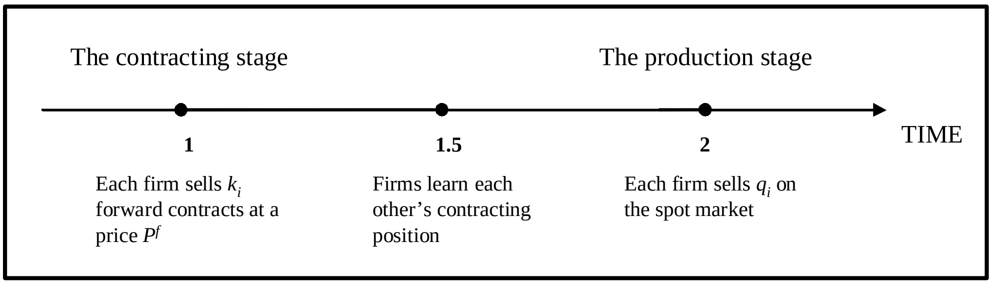

Fig.(1). Time schedule.

In the first period (i.e. *contracting stage*) each agent commits on its optimal forward trading volume's while in the second (i.e. *production stage*) it sets production decisions. Agents are symmetric, risk-neutral and rational.

Definition 1 *Let q_k and f_k denote the output and the trading volumes by the k-th firm (where $q_k \in \mathbb{R}^+$, $f_k \in \mathbb{R}$), and Q and F be the same at industry level (i.e. $Q = q_i + q_j$ and $F = f_i + f_j$, $k = i, j$).*

Spot market. Concerning the operation of the commodity market, the following is assumed. First, consistently with the literature on the subject, the inverse demand function is $P(Q)$ where $P_Q(Q) < 0$ and $P_{QQ}(Q) = 0$[11]. Second, competition in the commodity market is modeled by means of conjectural variations. There is a widespread recognition that such methodological framework, which is rather new in AV settings, is suitable to conduct robustness investigations and to estimate the likely impact of alternative concentration levels on equilibria, which is one purpose of the present study.[12] The core idea underlying the approach is as follows. Each operator is assumed to believe that any increase in its own production levels would lead to a constant and perfectly predictable output change by the rival. Formally

$$\frac{\partial q_{-k}(q_k)}{\partial q_k} \equiv \frac{\partial q_k(q_{-k})}{\partial q_{-k}} \equiv \alpha \tag{1}$$

where $\alpha \in \{-1; 0\}$, $k, -k = i, j$ and $k \neq -k$. Interestingly price-taking and price-setting attitudes are consistent with different values of the parameter. In particular $\alpha = -1$ and $\alpha = 0$ correspond to perfectly competitive (i.e. Bertrand-like) and Cournot-like behaviours respectively.[13]

[11] We define $F_x(x, y)$ and $F_y(x, y)$ as the (partial) derivatives of the function $F(x, y)$. Analogously, second derivatives are abbreviated.

[12] See for instance Bushnell, forthcoming.

[13] For a survey about the argument see Dixit (1986).

Forward market. We conform to the following couple of conventions. Agents are selling (buying) forward, or equivalently are taking a short (long) position in the contract market, whenever $f_k \geq 0$ ($f_k < 0$). We assume that both financial and physical delivery of the underlying commodity is possible and that the let the only relevant difference among the two is the non-negativity constraint couched in physical contracts where agents are required to produce at least the pre-committed level of output (i.e. $q_k - f_k \geq 0$).[14]

Concerning the operation of the forward market, the following is assumed. First, contracts positions are binding and observable before the spot market opens. Second, demand and competition modes in the contract market are similar (in form) to those introduced just above. The inverse demand function is $P^f(F)$ (where $P^f_F(F) < 0$ and $P^f_{FF}(F) = 0$) and conjectural variations are used to model competition

$$\frac{\partial f_{-k}(f_k)}{\partial f_k} \equiv \frac{\partial f_k(f_{-k})}{\partial f_{-k}} \equiv \beta \tag{2}$$

where $\beta \in \{-1; 0\}$, $k, -k = i, j$ and $k \neq -k$.

Technological structure of the industry: cost functions. Let's consider the technological structure of the industry. First, the production cost function, $C(q_k)$, is increasing and strictly convex in its argument (i.e. $C_q(q_k) > 0$ and $C_{qq}(q_k) > 0$). Remind that standard results are obtained for linear cost structures (i.e. $C_{qq}(q_k) = 0$). Quadratic costs functions, however, are required to get explicit solutions with conjectures. We return on the argument later when discussing specific formalizations. Second, each agent is aware that the submission of f_k forward contracts will cost (or pay back) him the amount in (3)

$$\left[P^f(F) - P(Q)\right] f_k \tag{3}$$

where in the first stage of the game $P(Q)$ should be intended as the opportunity cost of pre-commitments.

Perfect arbitrage condition. In the proceeding, next to the traditional (*ex-ante*) perfect arbitrage condition stating that forward and spot prices are efficiently arbitraged at the opening of the contracting stage, an *ex-post* equivalent is set where the same result (i.e. perfect arbitrage between forward and spot prices) is assumed to occur at the closure

[14] See, among the others, Joskow and Tirole (2000) for a review about the difference between financial and physical contracts.

of the forward market. The couple of conditions formalize as follows

$$\underbrace{P^f(F) - P(Q^*)}_{ex-ante} \quad \text{and} \quad \underbrace{P^f(F^*) - P(Q^*)}_{ex-post} \tag{4}$$

where the superscript '*' is used to denote equilibrium variables. Although both statements entail convergence between spot and forward price dynamics and are therefore consistent with empirical evidence, they hinder dissimilar trends.

Definition 2 *Total profit serves as objective function of the duopolists. Under the assumptions given above it results as follows*

$$\Pi^k(q_k, q_{-k}, f_k) = \underbrace{P(Q)q_k - C(q_k)}_{(a)} + \underbrace{\left[P^f(F) - P(Q)\right] f_k}_{(b)} \tag{5}$$

where (a) is the spot market component of profits while (b) is the forward equivalent.

4.1 The forward/spot interactions: standard results

This section provides some preliminary insights on *Nash* equilibria and related properties. Further remarks are presented in Appendix B where functional specifications are introduced and explicit solutions stated. Consistently with AV and the literature on the argument, the game is resolved backward. Spot market equilibrium is obtained at first so that the optimal forward commitment is set by anticipating the second stage best strategies' of producers.

Production stage. In the spot market each profit maximizing operator chooses production levels given (1) the conjectures on its rival's behaviour (i.e. $q_{-k}(q_k)$) and (2) forward market equilibria (i.e. f_k, f_{-k}, $P^f(F)$). Formally we have

$$q_k^* = \underset{q_k}{argmax}\ \Pi^k\left(q_k \middle| q_{-k}(q_k), f_k, f_{-k}\right). \tag{6}$$

Proposition 1 *Assuming a stable interior solution of the problem in (6) exists, q_k^* turns out as a function of predetermined variables and competition modes:*

$$q_k^* = q_k\left(\underset{+}{f_k}, \underset{-}{f_{-k}}, \underset{+/-}{\alpha}\right). \tag{7}$$

Prop.(1) has many interesting insights. We review them briefly. First, spot *Nash* equilibria are independent of forward market competition levels, which are measured by β in our simplified framework. We return on the argument later, demonstrating that in AV settings conjectures on forward interaction are irrelevant to final equilibria. All that matter is the degree of competition in the spot market and the technological structure of the industry. Second, excluding Bertrand competition in the commodity stage (i.e. $\alpha \neq -1$), rival's contracting position has a negative indirect impact on production decisions (i.e. $\partial q_k^*/\partial f_{-k} < 0$) while the opposite occurs with respect to its own contracting position (i.e. $\partial q_k^*/\partial f_k > 0$). The main intuition is as follows. Producers committing forward are less interested in high spot prices because of the need to refund buyers of contracts for the difference between strike and spot prices (see the forward component in profits as detailed in (5)), and behave more aggressively in the production stage (i.e. their reaction function moves outwards). The effect is based on the same intuition as the Coase's conjecture[15] which states that a monopolist producing a durable good does not succeed in setting the output price above its marginal cost. Third, price competition (i.e. $\alpha = -1$) is a necessary and sufficient condition to separate spot and forward decisions (in such case $(\partial q_i^*/\partial f_i) = (\partial q_i^*/\partial f_j) = 0$). This finding is due to the perfect substitutability of the commodity delivered in the two markets. Fourth, according to eq.(37), toughening competition in the spot market leads to reduction in the aggregate output whenever

$$q_k^* < f_k \quad \forall k = i, j. \tag{8}$$

We would remark that the inequality in (8) is inconsistent with the delivery of physical contracts where agents are required to produce at least the pre-committed level of output (i.e. $q_k \geq f_k$). However it may occur with financial commitments.

Contracting stage. After having derived the solution to the second stage of the game, we return to first period where the industry commit forward by (1) anticipating the consequences on the second stage (e.g. $q_k^* = q_k(f_k, f_{-k}, \alpha)$ $\forall k, -k = i, j$ and $k \neq -k$), (2) recognizing that equilibrium prices must fulfil the *ex-ante* perfect arbitrage condition (i.e. $P^f(F) = P(Q^*)$) and (3) given the conjectures on rival's behaviour (i.e. $f_{-k}(f_k)$). The

[15] For further details on the argument see Coase (1972).

first-stage optimization problem formalizes as follows

$$\begin{aligned} f_k^* = \quad & \arg\max_{f_k} \Pi^k \left(f_k | \, f_{-k}\left(f_k\right), q_k^*, q_{-k}^* \right) \\ & s.v. \; P^f(F) - P\left(Q^*\right) = 0, \end{aligned} \tag{9}$$

and gives a system ($k, -k = \{i, j\}$ and $k \neq -k$) of (symmetric) best replies

$$P_Q(Q) \left\{ \left(\frac{\partial q_{-k}^*}{\partial f_k} - \alpha \frac{\partial q_k^*}{\partial f_k} \right) q_k^* + (1+\alpha) \frac{\partial q_k^*}{\partial f_k} f_k \right\} = 0. \tag{10}$$

We can now find out the rationales for the sensitivity of AV outcomes (i.e. forward contracts as suitable means for mitigating dominant attitude in commodity markets) to spot market competition. We would preliminary remark that q_k^* is linear in forward commitments (i.e. $\partial q_k^* / \partial f_k$ and $\partial q_k^* / \partial f_{-k}$ are fixed). When assuming *price setting* attitudes in the spot market (i.e. $\alpha = -1$), the FOCs collape to zero[16] and therefore forward equilibria rest undetermined. However, assuming product differentiation and Bertrand competition, as in MS, we would obtain

$$\Pi_f^k(\cdot) = P_Q(\cdot) \left[\left(\partial q_{-k}^* / \partial f_k \right) + \left(\partial q_k^* / \partial f_k \right) \right] q_k^* < 0, \tag{11}$$

operators find it optimal to take long positions on the contract market (i.e. $f_k^* < 0$). Spot market demand moves outward because of the additional forward supply by producers. And since output decisions stand still (we have proved that Bertrand competition separates forward and spot market equilibria), the industry succeed in escaping the perfectly competitive outcome. Equivalently, forward commitments entail anti-competitive effects, which is the main finding of MS.

Let's revert to *Cournot competition* (i.e. $\alpha = 0$). The first order condition simplifies as follows

$$\frac{\partial q_{-k}^*}{\partial f_k} q_k^* + \frac{\partial q_k^*}{\partial f_k} f_k = 0 \quad \text{or equivalently} \quad f_k = -\frac{\partial q_{-k}^* / \partial f_k}{\partial q_k^* / \partial f_k} q_k^* \ , \tag{12}$$

where $f_k > 0$ because $\partial q_{-k}^* / \partial f_k < 0$, $\partial q_k^* / \partial f_k > 0$ and $q_k^* > 0$. Operators sell forward contracts at equilibrium and the pro-competitive impact of pre-commitments (i.e. AV) is "re-established". In the following, some static analyses are conducted.

Proposition 2 *Assuming a stable interior solution of the problem in (9) exists and the perfect arbitrage condition is met; f_k^* turns out as a function of spot market conjectures only. Formally*

$$f_k^* = f_k(\alpha) \quad k = \{i, j\}. \tag{13}$$

[16] Rearranging the terms in (10) under (*ex-ante*) perfect arbitrage we get: $\Pi_f^k(\cdot) = P_Q(\cdot)\left[\left(\partial q_{-k}^* / \partial f_k\right) + \left(\partial q_k^* / \partial f_k\right)\right] q_k^* = 0$

Prop.(2) points out an interesting insight. Because of perfect arbitrage, equilibria are functions of technological parameters and spot market conjectures only.

5 The forward/spot interaction with ex-post perfect arbitrage

This section investigates the robustness of the findings in Sec.(4) with respect to the perfect arbitrage condition. By altering the time distribution of the constraint (i.e. *ex-post* in spite of *ex-ante*), we prove that, notwithstanding the independence of final equilibria to β, there is a range of parameter values in order for manipulation in the forward market to be possible and such that operators find it optimal for exploitation. The full game is resolved by backward induction. As explained in the Introduction, *ex-post* arbitrage is possible whenever demand in the contract market is not exactly the residual from the spot market, de facto separating the two trading platforms. Profit opportunities in the contract market could be exploited, but they are limited by price convergence between spot and forward markets.

The second stage (i.e. *production stage*) is the same we have analyzed in Sec.(4). The proceeding provides a detailed formalization of the forward game. The superscript '^{s}' denotes market equilibria.

Contracting stage. In the first stage of the full game each agent commits forward by (1) anticipating the consequences on the second stage (e.g. $q_k^s = q_k^* = q_k(f_k, f_{-k}, \alpha)$ $\forall k, -k = i, j$ and $k \neq -k$), (2) recognizing that equilibrium prices must fulfil the *ex-post* perfect arbitrage condition (i.e. $P^f(F^s) = P(Q^s)$) and (3) given the conjectures on rival's behaviour (i.e. $f_{-k}(f_k)$). Notwithstanding the analogies with the maximization in (9), the solution to the constrained optimization in (14)

$$\begin{aligned} f_k^s = \quad & \arg\max_{f_k} \Pi^k \left(f_k | \, f_{-k}(f_k), q_k^s, q_{-k}^s \right) \\ & s.v. \; P^f(F^s) - P(Q^s) = 0 \end{aligned} \tag{14}$$

entails several relevant differences. The first stage system of best reply for each firm ($k = \{i, j\}$) is

$$P_Q(Q^s) \left[\left(\frac{\partial q_{-k}^s}{\partial f_k} - \alpha \frac{\partial q_k^s}{\partial f_k} \right) (q_k^s - f_k + \lambda_k) + (1+\alpha) \frac{\partial q_k^s}{\partial f_k} \lambda_k \right] + (1+\beta) P_F^f(F)(f_k - \lambda_k) = 0, \tag{15}$$

where λ_k is the Lagrange multiplier associated to the *ex-post* efficient arbitrage constraint. We have noticed in the introduction that λ_k would be the shadow value of the foregone profit in the contract market if intertemporal differentiation were allowed, and represents the share of contracts that a firm would sign to manipulate forward markets. The sign is endogenous (i.e. $\lambda_k \gtreqless 0$) and depends on the underlying framework (i.e. demand and cost parameters).

In Sec.(4) we have demonstrated that short positions (i.e. $f_k \geq 0$) prompt the aggressiveness of producers and succeed in alleviating the inefficiency of dominant attitudes while long positions (i.e. $f_k < 0$), by withholding capacity, exacerbate market power. Therefore in standard settings the suitability of forward contracts as mitigating devices depends on the optimal strategy by producers. In *ex-post* modelling frameworks, two core issues are at stake: (1) what is the likelihood of buying (selling) attitudes and (2) how many contracts the industry is submitting. While the former investigation gives us some insights on the effectiveness of forward commitments as a mitigating device, the latter explains the awaited efficiency of the mechanism as the larger the contract coverage, thus the closer the price is to marginal costs.

The first consequence of *ex-post* arbitrage is that long contracts can arise at equilibrium, an issue overlooked by AV theory.

Proposition 3 *(Sufficient) The industry takes short positions in the contract market whenever* $\left(\partial q^s_{-k}/\partial f_k\right) \leq \alpha \left(\partial q^s_k/\partial f_k\right)$ *and* $\lambda^s_k \geq 0$.

Prop.(3) demonstrates that *ex-post* perfect arbitrage does not exclude circumstances for forward commitments to effectively serve as a pro-competitive mechanism. Before deriving the conditions for manipulation in forward markets to be profitable, we analyse the impact of market power in forward markets (i.e. $P(Q) \neq P^f(F)$).

Lemma 1 *Assume a stable interior solution of the problem in (14) exists.* $f^s_k|_{\lambda^s_k=0}$ *(i.e. unconstrained) turns out as a function of the technological structure of the industry as well as of spot/forward competition modes, while* $f^s_k|_{\lambda^s_k\neq 0}$ *(i.e. actual) is independent of forward market competition. Formally*

$$f^s_k|_{\lambda^s_k=0} = f_k\left(\alpha,\beta\right). \quad \text{and} \quad f^s_k|_{\lambda^s_k\neq 0} = f_k\left(\underset{-}{\alpha}\right) \tag{16}$$

According to Lem.(1), market equilibria are functions of technological and strategic parameters (at least spot market competition). When the *ex-post* perfect arbitrage condition strictly binds (i.e. $\lambda_k^s \neq 0$), uncompetitive behaviours in the forward market do not affect market equilibria (i.e. $\partial f_k^s/\partial\beta = \partial q_k^s/\partial\beta = \partial q_{-k}^s/\partial\beta = 0$). However we do not return to standard dynamics (i.e. $\partial f_k^s/\partial\alpha \neq \partial f_k^*/\partial\alpha$). In particular notice, as long as production is decreasing in the degree of spot market concentration (i.e. $\partial Q^s/\partial\alpha < 0$), forward contracting moves the same way (eq.(44)).

The section concludes with some analyses on the profitability of forward market manipulation.[17] The question is as follows. Since, according to Lem.(1), equilibria rest independent of forward market competition modes, strategic behaviour in such market is really profitable?

Proposition 4 *(Cournot). Under quantity competition (i.e. $\alpha = 0$), a necessary and sufficient condition for manipulation of forward markets to be profitable (i.e. $\Pi^k\left(q_k^s, q_{-k}^s\right) \geq \Pi^k\left(q_k^*, q_{-k}^*\right)$, $\lambda_k^s \neq 0$) is*

$$f_k^* \geq f_k^s \; \forall k = \{i, j\}\,. \tag{17}$$

Corollary 1 *(Bertrand). Under price competition manipulation of forward market is unprofitable.*

Some remarks under functional specifications. To better investigates contracting with *ex-post* perfect arbitrage we let (1) the inverse demand functions be $P(Q) = A - q_i - q_j$ (where $P_Q(Q) = -1$ and $A > c$) in spot and $P^f(F) = B - f_i - f_j$ (where $P_F^f(F) = -1$ and $B > 0$) in contract markets respectively, and (2) the production cost function be $C(q_k) = (c/2)\,q_k^2$ (where $c > 0$, $C_q(q_k) = cq_k$ and $C_{qq}(q_k) = c$).

Notice we rely upon quadratic cost functions because homogeneity in the delivered commodity yields multiple (infinite) equilibria when combined with the traditional, linear cost structures in Bertrand settings.[18]

[17] Notice profitability of strategic behaviour in the contracting stage is measured as difference between *ex-ante* and *ex-post* (actual) profits. Formally

$$\Pi\left(q_i^s, q_j^s\right) \geq \Pi\left(q_i^*, q_j^*\right)$$

[18] To by-pass the matter in linear cost structures, MS have considered differentiated, instead of homogeneous, commodities.

Consider price competition at first. Since output and therefore spot market prices are independent of forward market equilibria, when the perfect arbitrage condition binds *ex-ante*, perfectly competitive - Bertrand-like - duopolists are neutral to the amount of pre-committed contracts (i.e. $f_k^* \gtreqless 0$). Results do not hold in contexts where the same constraint is assumed to bind *ex-post*. In fact, notwithstanding the independence of the second stage decisions, we find out that a given amount of forward is delivered at *ex-post* equilibrium (i.e. $f_k^s = \lambda_k^s \neq 0$). The rationale is in the potential for arbitrage profits which increase the appealingness of taking long positions. Equivalently the profitability of capacity withholding strategies diminishes. However, also notice that since the perfect arbitrage condition must verify at equilibrium, potential profits are not fully realized; also, depending on the parameters values, f_k^s can be positive or negative. In particular the first stage best replies for firm i in *ex-ante* and *ex-post* frameworks are as follows (firm j being symmetric):

$$\underbrace{P^f - \left(A - q_i^* - q_j^*\right) = 0}_{ex\text{-}ante} \quad \text{versus} \quad \underbrace{(1+\beta)\left(f_i^s - \lambda_i^s\right) = 0}_{ex\text{-}post \text{ (potential)}} \quad \underbrace{B - f_i^s - f_j^s - \frac{Ac}{(2+c)} = 0.}_{ex\text{-}post \text{ (actual)}} \tag{18}$$

The traditional *ex-ante* perfect arbitrage conditions results from the equality between price equilibrium in the second stage of the game (spot market competition) with $(Ac)/(2+c) = A - q_i^* - q_j^* = P(Q^*)$. The *ex-post* potential is obtained from the FOCs in eq.(15) with $\alpha = -1$, recalling that in Bertrand setting $\left(\partial q / \partial f_k\right) = \left(\partial q_{-k}^s / \partial f_k\right) = 0$ (see Prop.(1)). The *ex-post* actual gives the value of $\lambda_k^s(\beta)$ such that, when the constraint binds, the first stage best reply becomes the "market clearing condition" $P^f(F^s) = B - f_i^s - f_j^s = P(Q^s)$. Therefore, λ_k^s is the shadow value of the foregone profit in the contract market, if intertemporal differentiation were allowed. It can be also be interpreted as an endogenous discount factor between the spot and the contract market. Those marginal profit opportunities depend on the degree of competition in the forward market (β), but they vanish at equilibrium where $P^f(F^s) = P(Q^s)$ and $f_k^s|_{\lambda_k^s \neq 0} = f_k(\alpha)$ (see Lem.(1)). However, we know from Corollary 1 that these contracts do not increase profits, they only serve the forward demand.

Now let's revert to quantity competition, which is AV framework. We would preliminary remark the following. The findings discussed hereafter differ from those presented in AV because of quadratic cost functions.[19]

[19] See Appendix B for a detailed comparison between *ex-ante* (AV) and *ex-post* (actual) settings with linear cost structures.

The solution to the second-stage of the game confirms that in both *ex-ante* and *ex-post* models output market equilibria are affected by forward pre-commitments. As in the case of Betrand competition, differences in the operation of the perfect arbitrage constraint (i.e. *ex-ante* versus *ex-post*) alter the responsiveness of the setting to forward market equilibria. The first stage best replies for firm i in *ex-ante* and *ex-post* frameworks are provided below:

$$\underbrace{f_i^* = \frac{(1+c)\,A - f_j^*}{(2+c)\psi_0}}_{ex\text{-}ante} \tag{19}$$

versus

$$\underbrace{f_i^s = -\frac{1}{\psi_1} f_j^s + \frac{(1+c)(A-\lambda_i^s)}{\psi_2} + \frac{(c+3)^2(c+1)^2(\beta+1)}{\psi_2}\lambda_i^s;}_{ex\text{-}post\ \text{(potential)}} \quad \underbrace{f_i^s = -f_j^s + \frac{(B(5+3c)-2A(1+c))}{(2+c)}}_{ex\text{-}post\ \text{(actual)}}$$

where $\psi_0 = 4c+c^2+2$, $\psi_1 = \left[(\beta+1)\left(4c+c^2+3\right)^2 + \psi_0\right]^{-1}$ and $\psi_2 = (c+3)^2(c+1)^2(\beta+1)+\psi_0$.

Except for a proper combination of parameter values (i.e. A, B, c) not only the steepness but also the intercepts of the three above differ. As noticed when assuming price competition, *ex-post* perfect arbitrage expands the attractiveness of forward markets. However, since the constraint is assumed to bind at equilibrium, agents must review their purposes to assure convergence between spot and forward prices. As in Bertrand competition, the values of λ_i^s, obtained by solving the system of equations given by the constraint and the first order conditions of the second stage maximization problem, are a function of β. The potential gain in exerting market power in the contract market shrinks at the constrained equilibrium. This results in the independence of the realized best replies from forward market conjectures, that is from β. By invoking symmetry, one obtains optimal contracting in Cournot setting, $f^* = A/\left(5c+c^2+5\right)$, as well as in the ex post settings,$f^s = \frac{1}{2(c+2)}\left(B(5+3c) - 2A(1+c)\right)$. In this latter case, it is possible for firm to sign long contracts (that is when $A > \frac{B(3c+5)}{2(c+1)}$).

6 The forward/spot interaction with stackelberg leadership

Together with the likelihood of strategic behaviours in the contract stage, a further relevant evidence in the electricity sector is the occurrence of strategic leadership in the spot market, which is the issue analyzed hereafter. Notice leader and follower differ because of asymmetric cost structures (i.e. agent face asymmetric production cost functions: $C^i(q_i) \neq C^j(q_j)$). Notwithstanding the simultaneous submission of forward contracts by each player, and without any loss of generality, we assume that agent i is a first mover on the spot market. Not surprisingly, the leader and the follower face asymmetric maximization problems and this leads to asymmetric optimal strategies at equilibrium. It is usually argued that the leader exploits its first mover advantage to sustain spot prices. In the proceeding we detail the functioning of such tactic.[20]

The proceeding is organized as follows. Through backward induction we study the best replies in the spot market by the follower and subsequently by the leader and incorporate such optimal strategic behaviours in the selection of the optimal simultaneous first stage forward commitment.

To avoid unnecessary repetitions, the superscript ’^{l}’ is used to denote equilibrium variables.

Production stage. In the commodity game the follower, j, sets output given (1) the leader's optimal production level (i.e. q_i) and (2) forward market equilibria (i.e. f_i, f_j, $P^f(F)$). The maximization problem formalizes in (20)

$$q_j^l = \underset{q_j}{argmax}\ \Pi^j\left(q_j \middle| q_i, f_i, f_j\right) \tag{20}$$

and the solution to the FOC yields the optimal strategy by the follower as a function of q_i and f_j (i.e. $q_j^l = q_j(q_i, f_j)$) where

$$\frac{\partial q_j^l}{\partial q_i} = -\frac{P_Q(\cdot)}{2P_Q(\cdot) - C_{qq}(\cdot)} < 0 \quad \text{and} \quad \frac{\partial q_j^l}{\partial f_j} = \frac{P_Q(\cdot)}{2P_Q(\cdot) - C_{qq}(\cdot)} > 0. \tag{21}$$

According to the inequalities in (21), producer j reacts symmetrically to changes in its determinants (i.e. $\partial q_j^l/\partial q_i = -\partial q_j^l/\partial f_j$), which may be intended as strategic substitutes. Intuitively the follower commits forward to counterbalance the first mover advantage by the leader. We return on the issue when studying the first stage of the full forward/spot interaction. In addition notice f_i has no direct impact on q_j^l.

Let's consider the best reply by the Stackelberg leader. The first mover chooses its optimal production level given (1) $q_j^l = q_j(q_i, f_j)$, i.e. anticipating the best reply by agent

[20] It is worth to notice that because of the sequential structure of spot market interaction, the conjectural variation approach is no more appropriate.

j, and (2) forward market equilibria (i.e. f_i, f_j, $P^f(F)$). The solution to the maximization problem in (22)

$$q_i^l = \underset{q_i}{argmax}\ \Pi^j \left(q_i | \, q_j^l, f_i, f_j \right), \tag{22}$$

yields the optimal production level by the leader as a function of forward market equilibria (i.e. $q_i^l = q_i(f_i, f_j)$), where

$$\frac{\partial q_i^l}{\partial f_i} = P_Q(\cdot)\frac{P_Q(\cdot)-C_{qq}^j()}{\Omega_3} \quad \text{and} \quad \frac{\partial q_i^l}{\partial f_j} = -\frac{\left[P_Q(\cdot)\right]^2}{\Omega_3}\ , \tag{23}$$

and $\Omega_3 = 2P_Q(\cdot)\left[P_Q(\cdot) - C_{qq}^j(\cdot)\right] - \left[2P_Q(\cdot) - C_{qq}^j(\cdot)\right]C_{qq}^i(\cdot)$.

Despite Stackelberg leadership in the spot game, forward contracts go on serving as a binding device. Short producers behave more aggressively in the output market not only for the refunding obligation but also because of the awaited increase in the market share.

Lemma 2 *Stackelberg leadership entails quantitative (not qualitative) differences compared to traditional simultaneous settings. Formally* $\partial q_k^l/\partial f_k > 0$ *and* $\partial q_k^l/\partial f_{-k} < 0$ *but* $\partial q_k^*/\partial f_k \neq \partial q_k^l/\partial f_k$ *and* $\partial q_k^*/\partial f_{-k} \neq \partial q_k^l/\partial f_{-k}$ *where* $k, -k = i, j$ *and* $k \neq -k$.

We disclose an interesting effect lacking in traditional models: spot market equilibria over-react to f_j.

Contracting stage. After having derived the solution to the second stage of the game, we return to the first period where the industry commits forward by (1) anticipating the consequences on the second stage (e.g. $q_i^l = q_i(f_i, f_j)$ and $q_j^l = \widetilde{q}_j(f_i, f_j)$); (2) recognizing that equilibrium prices must fulfil the perfect arbitrage condition (i.e. $P^f(F) = P(Q)$) and (3) given the conjectures on rival's behaviour (i.e. $f_{-k}(f_k)$). The first stage maximization problems by the leader (i.e. i) and the follower (i.e. j) are formalized below

$$f_i^l = \underset{f_i}{argmax}\ \Pi^i \left(f_i | \, f_j(f_i), q_j^l, q_i^l \right) \quad \text{and} \quad f_j^l = \underset{f_j}{argmax}\ \Pi^j \left(f_j | \, f_i(f_j), q_i^l, q_j^l \right)\ . \tag{24}$$

The solution to the system of best replies in (25) yields the optimal pre-commitment as a function of the technological structure of the industry

$$\begin{array}{c} P_Q(\cdot)\left\{\left[f_i + \frac{P_Q(\cdot)\left(q_i^l - f_i\right)}{2P_Q(\cdot) - C_{qq}^j(\cdot)}\right]\frac{\partial q_i^l}{\partial f_i} + q_i^l\frac{\partial q_j^l}{\partial f_i}\right\} = 0 \\ P_Q(\cdot)\left(f_j\frac{\partial q_j^l}{\partial f_j} + q_j^l\frac{\partial q_i^l}{\partial f_j}\right) = 0 \end{array}\ . \tag{25}$$

It is interesting to notice that while the follower takes a short position in the contract market (i.e. $f_j^l > 0$), the leader sells (buys) no contracts at equilibrium (i.e. $f_i^l = 0$). Since the quantity produced by the leader increases with his contracts, firm i succeeds in restricting the spot quantity by not participating to the contract market. Finally notice the best reply by the follower reveals partial coverage (i.e. $\left(\partial q_j^l / \partial f_j\right) > \left|\partial q_i^l / \partial f_j\right|$). Leaving away risk-aversion, the first mover advantage turns out as a substitute to forward pre-commitments, absent arbitrage opportunities. By converse, the follower submits contracts to counterbalance rival's leadership in the spot game. Interestingly, the combination of the two effects (i.e. leadership advantage and $f_i^l = 0$) may return in the follower getting a higher share of profits. In fact, it is widely recognized that unilateral contracting yields the committing firm an advantage similar to the one it would attain under a quantity leadership.

6.1 The forward/spot interaction with stackelberg leadership and ex-post perfect arbitrage

Applying the same methodology of Sec.(5), in the proceeding we investigate the robustness of previous findings to the perfect arbitrage condition. We have proved in standard settings that by altering the time distribution of the constraint (i.e. *ex-post* in spite of *ex-ante*), manipulation of forward prices may be profitable and operators exploit it.

The spot market competition is the same we have analyzed in Sec.(6). The proceeding provides a detailed formalization of the forward stage. To avoid unnecessary repetitions, the superscript $'^{lp}'$ denotes equilibrium variables.

Contracting stage. In the first stage of the full game each agent commits forward by (1) anticipating the consequences on the second stage (e.g. $q_i^{lp} = q_i^l = q_i(f_i, f_j)$ and $q_j^{lp} = q_j^l = \widetilde{q}_j(f_i, f_j)$), (2) recognizing that equilibrium prices must fulfil the *ex-post* perfect arbitrage condition (i.e. $P^f(F^{lp}) = P\left(Q^{lp}\right)$) and (3) given the conjectures on rival's behaviour (i.e. $f_{-k}(f_k)$). The FOCs to constrained maximization problems in (26)

$$
\begin{aligned}
f_i^{lp} = \ & \arg\max_{f_i} \Pi^i \left(f_i | \, f_j(f_j), q_i^{lp}, q_j^{lp} \right) \qquad & f_j^{lp} = \ & \arg\max_{f_j} \Pi^j \left(f_j | \, f_i(f_j), q_i^{lp}, q_j^{lp} \right) \\
& s.v. \ P^f(F^{lp}) - P\left(Q^{lp}\right) = 0 & & s.v. \ P^f(F^{lp}) - P\left(Q^{lp}\right) = 0
\end{aligned}
\tag{26}
$$

are

$$P_Q(\cdot)\left(\frac{\partial q_j^{lp}}{\partial f_i}+\frac{\partial q_i^{lp}}{\partial f_i}\right)\lambda_i+(1+\beta)\,P_F^f(\cdot)\,(f_i-\lambda_i)=0$$

$$P_Q(\cdot)\left(\frac{\partial q_j^{lp}}{\partial f_j}+\frac{\partial q_i^{lp}}{\partial f_j}\right)\left[\frac{\partial q_i^{lp}}{\partial f_j}\left(q_j^{lp}-f_j\right)+\left(\frac{\partial q_j^{lp}}{\partial f_j}+\frac{\partial q_i^{lp}}{\partial f_j}\right)\lambda_j\right]+(1+\beta)\,P_F^f(\cdot)\,(f_j-\lambda_j)=0 \tag{27}$$

where λ_i, λ_j are the Lagrange multipliers of the *ex-post* efficient arbitrage constraints.

As long as the suitability of forward contracts as mitigating devices rests upon the optimal strategy by producers, the proceeding is organized around two core issues, underpinned by some comparative static analyses. First, we focus on the effect of *ex-post* perfect arbitrage on buying (selling) attitudes. Second, to assess the likelihood of strategic behaviours in the contract stage, we compare *ex-post* and *ex-ante* frameworks with respect to related profitabilities.

Proposition 5 *(Sufficient) Each operator in the industry takes a short position in the contract market (i.e.* $f_k^{lp}\geq 0,\ k=\{i,j\}$*) if*

$$\left\{\left(1>\tau\frac{\partial Q^{lp}}{\partial f_i}\right)\vee\left(\lambda_i^{lp}>0\right)\right\}\wedge\left\{\left(1>\tau\left(\frac{\partial Q^{lp}}{\partial f_j}\right)^2\right)\vee\left(\lambda_j^{lp}\geq 0\right)\right\} \tag{28}$$

where $\tau=P_Q\left(\cdot\right)(1+\beta)^{-1}\left[P_F^f(\cdot)\right]^{-1}$.

Because of *ex-post* perfect arbitrage, the likelihood of both short and long positions depends on the steepness of (forward and spot) demands as well as on the responsiveness of aggregate production to forward commitments (i.e. $\partial Q^{lp}/\partial f_k$). Excluding convergence in prices (i.e. $P^f(F^{lp})\neq P\left(Q^{lp}\right)$) *id est* focusing on unconstrained outcomes, the leader would return to standard results

$$\left.f_i^{lp}\right|_{\lambda_i^{lp}=0}=\left[1-\tau\frac{\partial Q^{lp}}{\partial f_i^{lp}}\right]\left.\lambda_i^{lp}\right|_{\lambda_i^{lp}=0}=f_i^l=0 \tag{29}$$

and the follower would sell forward

$$\left.f_j^{lp}\right|_{\lambda_j^{lp}=0}=-\frac{\mu}{(1+\beta)P_F^f(\cdot)-\mu}q_j^{lp}+P_Q\left(\cdot\right)\frac{1-\tau\left(\partial Q^{lp}/\partial f_j\right)^2}{P_Q(\cdot)-\mu\tau}\left.\lambda_j^{lp}\right|_{\lambda_j^{lp}=0}>0. \tag{30}$$

Lem.(3) provides some comparative static analyses.

Lemma 3 *Assume a stable interior solution of the problem in (26) exists.* $\left.f_j^{lp}\right|_{\lambda_j^s=0}$ *turns out as a function of the technological structure of the industry and of forward competition modes, while* $\left.f_i^{lp}\right|_{\lambda_i^s=0}=\left.f_k^{lp}\right|_{\lambda_k^s\neq 0}$ *are independent of forward market behaviours.*

The section concludes with some analyses on the profitability of forward market manipulation[21]. The question is as follows. Since, according to Lem.(3), *ex-post* equilibria are independent of forward market competition modes, do startegic behaviours in such market be really profitable?

Proposition 6 *A necessary and sufficient condition for manipulation of forward markets to be profitable (i.e.* $\Pi^k\left(q_i^{lp}, q_j^{lp}\right) \geq \Pi^k\left(q_i^l, q_j^l\right)$, $\lambda_k^{lp} \neq 0$*) is*

$$f_k^l \geq f_k^{lp} \ \forall k = \{i, j\}\,. \tag{31}$$

Some remarks under functional specifications.

Remark 3 *(1) The inverse demand functions are* $P(Q) = A - q_i - q_j$ *(where* $P_Q(Q) = -1$ *and* $A > c$*) for the spot and* $P^f(F) = B - f_i - f_j$ *(where* $P_F(F) = -1$ *and* $B > 0$*) for the contract markets respectively. (2) The production cost function is* $C^i(q_i) = (c/2)\, q_i^2$ *for the leader and* $C^j(q_j) = 0$ *for the follower.*

Here we differentiate between leader and follower optimal contracting.

$$\underbrace{f_i^l = 0}_{ex\text{-}ante} \quad \text{versus} \quad \underbrace{f_i^{lp} = \lambda_i^{lp}\left(\frac{1}{4}\frac{4\beta(c+1)+4c+3}{(c+1)(\beta+1)}\right)}_{ex\text{-}post \text{ (potential)}} \quad \underbrace{f_i^{lp} = \frac{4B(1+c)-A(1+2c)-f_j^{lp}(3+2c)}{4c+3}}_{ex\text{-}post \text{ (actual)}} \tag{32}$$

$$\underbrace{f_j^l = \frac{A(1+2c)-f_i^l}{(2c+3)(2c+1)}}_{ex\text{-}ante} \tag{33}$$

versus

$$\underbrace{f_j^{lp} = \frac{A(1+2c)-f_i^{lp}+2\lambda_j(c+1)(2c+4\beta(1+4c)+3)}{9(1+2c)+8(c^2+\beta(c+1)^2)}}_{ex\text{-}post \text{ (potential)}} \quad \underbrace{f_j^{lp} = \frac{4B(1+c)-A(1+2c)-f_i^{lp}(3+4c)}{2c+3}}_{ex\text{-}post \text{ (actual)}}$$

Under functional specifications several interesting insights emerge: the leader, who sustains spot market price by not selling contract in *ex-ante* setting, would jump to positive contracts ($f_i^{lp} = \lambda_i^{lp}\left(\frac{1}{4}\frac{4\beta(c+1)+4c+3}{(c+1)(\beta+1)}\right) > 0$ with $\beta \neq -1$) to exploit profit opportunities in

[21] Recall that profitability of startegic behaviour in the contracting stage is measured as difference between *ex-ante* and *ex-post* (actual) profits. Formally

$$\Pi^k\left(q_i^{lp}, q_j^{lp}\right) \geq \Pi^k\left(q_i^l, q_j^l\right)$$

the contract market due to inter-temporal differentiation. When the constraint is binding, the firm contracting position depends on demand and cost parameters, which determine the intensity of substitution between $f_i^{lp}$and f_j^{lp}. The follower, instead, to counterbalance the leader dominant position, is willing to become the sole supplier of contract when the arbitrage condition holds *ex-ante* ($f_j^l = \frac{A(1+2c)}{(2c+3)(2c+1)} > 0$). At the constrained equilibrium, his optimal amount of contracts can increase or decrease, depending on the parameters. Notice that the reaction functions *ex-post* are asymmetric, as a consequence of different spot quantities obtained as equilibrium of the spot market subgame. In particular, the sensitivity of the follower's reaction function to f_i^{lp} is larger, in absolute value, than the intensity of substitution shown in the leader's reaction function to f_j^{lp}. We show in Appendix B that solving the Stackelberg model with linear costs, price and profits may increase in *ex-post* settings compared to the equilibria calculated under the traditional non-arbitrage assumption. The price in the spot market is increased as total contracts decrease with respect to the equilibrium values calculated when the possibility of arbitrage is ruled out *ex-ante*.

7 Conclusions

This work provides a rationale for the economic and political paradoxes on the effectiveness of forward contracts in mitigating dominance and details the likely determinants of the dualistic nature of long-term commitments by means of a reinterpretation of existing models. Presenting those models in a common setting facilitates and enriches the task of understanding the alternative contributions, by comparing related findings. Moreover, it provides a detailed and unbiased rationale for supporting the inclusion (or exclusion) of long term contracts from the complete portfolio of policy measures that are intended to mitigate market power in strategic output markets.

From the regulatory and competition policy perspectives, our results suggest that regulating forward contracting levels has an additional spot market competitiveness benefit only if these purchases are structured as fixed-price forward contracts for fixed amount of energy, to avoid the possibility of inter-temporal gaming. As Kamat and Oren (2004) affirm, [...] "The limited amount of contracting in California (even after the ban by the California Public Utility Commission was removed) and the collapse of the California Power Exchange (PX) may suggest that some form of regulatory intervention (short of direct government purchases) might be required to insure that the public is protected by an adequate amount of forward contracts". Our analysis is the first attempt to understand as to how dominant firms manipulate contracts markets, an issue overlooked by the IO literature and closer to financial modelling. Further research in this direction includes modelling collusion *à la* Liski-Montero (2007), in order to fully understand how generators with market power on spot markets can influence forward prices in dynamic settings.

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A Appendix A

A.1 Proof of Proposition 1

The proof is straightforward. Necessary and sufficient first (FOCs) and second order conditions (SOCs) for having a stable interior solution of the problem in (6) are

$$\Pi_q^k(\cdot) = 0 \vee \Pi_{qq}^k(\cdot) < 0 \vee \Omega_1 = \Pi_{qq}^k(\cdot)\,\Pi_{qq}^{-k}(\cdot) - [P_Q(Q)]^2 > 0 \tag{34}$$

where $\{k, -k\} = i, j$ and $k \neq -k$. Therefore, by applying the implicit function theorem on the second-stage system of best replies (i.e. $\Pi_q^i(\cdot) = 0 \vee \Pi_q^j(\cdot) = 0$) we get

$$\frac{\partial q_i^*}{\partial f_i} = (1+\alpha)\,P_Q(\cdot)\,\frac{2(1+\alpha)P_Q(\cdot) - C_{qq}\left(q_j^*\right)}{\Omega_1}; \tag{35}$$

$$\frac{\partial q_i^*}{\partial f_j} = -(1+\alpha)\,\frac{\left[P_Q(\cdot)\right]^2}{\Omega_1}; \tag{36}$$

$$\frac{\partial q_i^*}{\partial \alpha} = -P_Q(\cdot)\frac{\left(q_i^* - f_i\right)\left[2(1+\alpha)P_Q(\cdot) - C_{qq}\left(q_j^*\right)\right] + P_Q(\cdot)\left(q_j^* - f_j\right)}{\Omega_1}. \tag{37}$$

Under the conditions in (34) and reminding that $\alpha \in \{-1; 0\}$, $P_Q(\cdot) < 0$ and $C_{qq}(\cdot) > 0$, the awaited dynamics verify (i.e. $\partial q_i^*/\partial f_i \geq 0$, $\partial q_i^*/\partial f_j \leq 0$ and $\partial q_i^*/\partial \alpha \gtreqless 0$). Results for agent j are symmetric.

A.2 Proof of Proposition 2

The proof is straightforward. Necessary and sufficient conditions (i.e. FOCs and SOCs) for having a stable interior solution of the problem in (9) are

$$\Pi_f^k(\cdot) = 0 \vee \Pi_{ff}^k(\cdot) < 0 \vee \Omega_2 = \Pi_{ff}^k(\cdot)\,\Pi_{ff}^{-k}(\cdot) - \left[P_Q(\cdot)\frac{\partial q_{-k}}{\partial f_k}\right]^2 \left(\frac{\partial q_{-k}}{\partial f_k} - \alpha\frac{\partial q_k}{\partial f_k}\right)\left(\frac{\partial q_{-k}}{\partial f_k} - \alpha\frac{\partial q_{-k}}{\partial f_{-k}}\right) > 0 \tag{38}$$

where $\{k, -k\} = i, j$ and $k \neq -k$. Therefore, by applying the implicit function theorem on the first stage system of best replies (i.e. $\Pi_f^i(\cdot) = 0 \vee \Pi_f^j(\cdot) = 0$) we get

$$\frac{\partial f_k^*}{\partial \alpha} = \frac{\left[P_Q(Q^*)\right]^2}{\Omega_2}\left\{\left[(1+\alpha)\frac{\partial q_k^*}{\partial f_k} + \vartheta\left(\frac{\partial q_k^*}{\partial f_k} - \frac{\partial q_{-k}^*}{\partial f_k}\right)\right]\left[\frac{\partial q_k^*}{\partial f_k}\left(q_k^* - f_k^*\right) - \vartheta\frac{\partial q_k^*}{\partial \alpha}\right]\right\} \tag{39}$$

where $\vartheta = \left(\partial q_{-k}^*/\partial f_k\right) - \alpha\left(\partial q_k^*/\partial f_k\right) \forall k = \{i, j\}$ and therefore $f_k^* = f_k(\alpha)$. In our generalized setting, notice that the final effect of increasing the degree of spot market competition on forward commitments is uncertain (i.e. $\partial f_k^*/\partial \alpha \gtreqless 0$). Functional specifications are required to get unambiguous dynamics.

A.3 Proof of Proposition 3

Rearranging the best replies in (15) we get

$$f_k^s = -\tfrac{P_Q(\cdot)\vartheta}{\delta - P_Q(\cdot)\vartheta} q_k^s + \lambda_k^s. \tag{40}$$

where $\delta = (1+\beta) P_F^f(\cdot) \leq 0$ and $\vartheta = \left(\partial q_{-k}^s / \partial f_k\right) - \alpha \left(\partial q_k^s / \partial f_k\right)$. Given the assumptions in Sec.(3) and recalling that $q_k^s > 0$, a sufficient condition for $f_k^s \geq 0$ is $(\vartheta \leq 0) \vee (\lambda_k^s \geq 0)$ *id est* $\left(\partial q_{-k}^s / \partial f_k\right) \leq \alpha \left(\partial q_k^s / \partial f_k\right)$ and $\lambda_k^s \geq 0$.

A.4 Proof of Lemma 1

The proof is straightforward. By applying the implicit function theorem on the system of FOCs in (15) under $\lambda_k^s = \lambda_{-k}^s = 0$ and $\lambda_k^s, \lambda_{-k}^s \neq 0$ we get

$$\left.\frac{\partial f_k^s}{\partial \beta}\right|_{\lambda_k^s = 0} = \tfrac{P_F^f(\cdot)}{\Omega_2} \left\{ f_{-k}^s \left[P_Q(\cdot)\vartheta \tfrac{\partial q_k^s}{\partial f_{-k}} \right] - f_k^s \left[P_Q(\cdot)\vartheta \left(\tfrac{\partial q_{-k}^s}{\partial f_{-k}} - 1 \right) + (1+\beta) P_F^f(\cdot) \right] \right\} \tag{41}$$

$$\left.\frac{\partial f_k}{\partial \alpha}\right|_{\lambda_k^s = 0} = \tfrac{P_Q(\cdot)}{\Omega_2} \left\{ \begin{array}{c} \left[\tfrac{\partial q_k^s}{\partial f_k} \left(q_k^s - f_k^s\right) - \vartheta \tfrac{\partial q_k^s}{\partial \alpha} \right] \left[P_Q(\cdot)\vartheta \left(\tfrac{\partial q_{-k}^s}{\partial f_{-k}} - 1 \right) + (1+\beta) P_F^f(\cdot) \right] \\ + P_Q(\cdot)\vartheta \tfrac{\partial q_k^s}{\partial f_{-k}} \left[\vartheta \tfrac{\partial q_{-k}^s}{\partial \alpha} - \tfrac{\partial q_{-k}^s}{\partial f_{-k}} \left(q_{-k}^s - f_{-k}^s\right) \right] \end{array} \right\} \tag{42}$$

$$\left.\frac{\partial f_k^s}{\partial \beta}\right|_{\lambda_k^s \neq 0} = 0 \tag{43}$$

$$\left.\frac{\partial f_k^s}{\partial \alpha}\right|_{\lambda_k^s \neq 0} = \frac{1}{2} \tfrac{P_Q(\cdot)}{P_F^f(\cdot)} \tfrac{\partial Q^s}{\partial \alpha} \tag{44}$$

where $\{k, -k\} = i, j$ and $k \neq -k$, $\Omega_2 = \Pi_{ff}^k(\cdot)\, \Pi_{ff}^{-k}(\cdot) - \left[P_Q(\cdot)\left(\partial q_k^s / \partial f_{-k}\right)\right]^2 \vartheta^2 > 0$, $\vartheta = \left(\partial q_{-k}^s / \partial f_k\right) - \alpha \left(\partial q_k^s / \partial f_k\right) = \left(\partial q_k^s / \partial f_{-k}\right) - \alpha \left(\partial q_{-k}^s / \partial f_{-k}\right)$ and hence

$$f_k^s|_{\lambda_k^s = 0} = f_k(\alpha, \beta) \quad \text{and} \quad f_k^s|_{\lambda_k^s \neq 0} = f_k\left(\underset{-}{\alpha}\right). \tag{45}$$

A.5 Proof of Proposition 4

E*x-ante* and *ex-post* (actual, $\lambda_k^s \neq 0$) equilibrium profits are

$$\Pi^k\left(q_k^*, q_{-k}^*\right) = P(Q^*) q_k^* - C(q_k^*) \quad \text{and} \quad \Pi^k\left(q_k^s, q_{-k}^s\right) = P(Q^s) q_k^s - C(q_k^s). \tag{46}$$

Assume $q_k^s = q_k^* + \varepsilon$, where $\varepsilon \approx 0$ and $k = \{i, j\}$. By using Tylor expansion (46) can be written as follows:

$$\Pi^k\left(q_k^*, q_{-k}^*\right) \simeq \Pi_q^k\left(q_k^s, q_{-k}^s\right)\left(q_k^* - q_k^s\right) + \frac{1}{2}\Pi_{qq}^k\left(q_k^s, q_{-k}^s\right)\left(q_k^* - q_k^s\right)^2 \tag{47}$$

$$\Pi^k\left(q_k^s, q_{-k}^s\right) \simeq \Pi_q^k\left(q_k^*, q_{-k}^*\right)\left(q_k^s - q_k^*\right) + \frac{1}{2}\Pi_{qq}^k\left(q_k^*, q_{-k}^*\right)\left(q_k^s - q_k^*\right)^2. \tag{48}$$

Adding the conditions for profit maximization (i.e. $\Pi_q^k\left(q_k^s, q_{-k}^s\right) = \Pi_q^k\left(q_k^*, q_{-k}^*\right) = 0$), profitability of forward market manipulation leads

$$\frac{1}{2}\left[\Pi_{qq}^k\left(q_k^s, q_{-k}^s\right) - \Pi_{qq}^k\left(q_k^*, q_{-k}^*\right)\right]\left(q_k^s - q_k^*\right)^2 \geq 0 \tag{49}$$

where $\Pi_{qq}^k\left(q_k, q_{-k}\right) = P_Q(Q) - C_{qq}(q_k)$. Recalling that spot market demand is downward sloping (i.e. $P_Q(\cdot) < 0$) and that costs are convex (i.e. $C_{qq}(\cdot) \geq 0$) we obtain

$$\left[P_Q(Q^s) - P_Q(Q^*)\right] - C_{qq}(q_k^s) + C_{qq}(q_k^*) \leq 0 \implies q_k^* > q_k^s \ . \tag{50}$$

Ineq.(50) must hold $\forall k = \{i, j\}$, hence

$$\textstyle\sum_{k=i,j} q_k^* = Q^* > \sum_{k=i,j} q_k^s = Q^s. \tag{51}$$

After some mathematics, from Prop.(1) we have that in the Cournot settings production is increasing in forward commitments

$$\left.\tfrac{\partial Q}{\partial f_k}\right|_{\alpha=0} = \tfrac{P_Q(\cdot)}{\Omega_1}\left[P_Q(\cdot) - C_{qq}(\cdot)\right] > 0 \tag{52}$$

therefore

$$\Pi^k\left(q_k^s, q_{-k}^s\right) \geq \Pi^k\left(q_k^*, q_{-k}^*\right) \implies q_k^* > q_k^s \implies Q^* > Q^s \implies f_k^* > f_k^s \tag{53}$$

and results are straightforward.

A.6 Proof of Corollary 1

We have demonstrated above that in Bertrand settings spot equilibria are independent of forward decisions (see Prop.(1)). In both *ex-ante* and *ex-post* models the following verifies

$$\left.\tfrac{\partial Q}{\partial f_k}\right|_{\alpha=-1} = 0, \tag{54}$$

and profits depend on spot decisions solely. Formally

$$\Pi^k\left(q_k^s, q_{-k}^s\right) \geq \Pi^k\left(q_k^*, q_{-k}^*\right) \Rightarrow q_k^* > q_k^s \ . \tag{55}$$

A.7 Proof of Lemma 2

The proof is straightforward. By assumption $C_{qq}^k(\cdot) > 0$ and $P_Q(\cdot) < 0$ therefore $\Omega_3 > 0$ and

$$q_i^l = q_i\left(\underset{+}{f_i}, \underset{-}{f_j}\right) \tag{56}$$

furthermore via the implicit function theorem we obtain

$$q_j^l = q_j\left(q_i, f_j\right) = \widetilde{q}_j\left(\underset{-}{f_i}, \underset{+}{f_j}\right) \tag{57}$$

where $\frac{\partial q_j^l}{f_i} = -\frac{P_Q^2\left(P_Q - C_{qq}^j\right)}{\left(2P_Q - C_{qq}^j\right)\Omega_3} < 0$ and $\frac{\partial q_j^l}{\partial f_j} = \frac{P_Q\left(\Omega_3 + P_Q^2\right)}{\left(2P_Q - C_{qq}^j\right)\Omega_3} > 0$.

Notwithstanding the similarity with AV and MS in overall patterns, they differ quantitatively. After some mathematics we get

$$\left.\frac{\partial q_i^*}{\partial f_i}\right|_{\alpha=0} > \frac{\partial q_i^l}{\partial f_i} > \left.\frac{\partial q_i^*}{\partial f_i}\right|_{\alpha=-1} \quad \text{and} \quad \left|\frac{\partial q_i^l}{\partial f_j}\right| > \left|\min\left\{\left.\frac{\partial q_i^*}{\partial f_j}\right|_{\alpha=-1}; \left.\frac{\partial q_i^*}{\partial f_j}\right|_{\alpha=0}\right\}\right| \tag{58}$$

$$\frac{\partial q_j^l}{\partial f_j} > \max\left\{\left.\frac{\partial q_j^*}{\partial f_j}\right|_{\alpha=-1}; \left.\frac{\partial q_j^*}{\partial f_j}\right|_{\alpha=0}\right\} \quad \text{and} \quad \left|\left.\frac{\partial q_j^*}{\partial f_i}\right|_{\alpha=0}\right| > \left|\frac{\partial q_j^l}{\partial f_i}\right| > \left|\left.\frac{\partial q_j^*}{\partial f_i}\right|_{\alpha=-1}\right|. \tag{59}$$

A.8 Proof of Proposition 5

By rearranging the best replies in (27) we get

$$\begin{aligned} f_i^{lp} &= \left[1 - \tau\frac{\partial Q^{lp}}{\partial f_i^{lp}}\right]\lambda_i^{lp} \\ f_j^{lp} &= -\frac{\mu}{(1+\beta)P_F^f(\cdot) - \mu} q_j^{lp} + P_Q(\cdot)\frac{1 - \tau\left(\partial Q^{lp}/\partial f_j\right)^2}{P_Q(\cdot) - \mu\tau}\lambda_j^{lp} \end{aligned} \tag{60}$$

where $\left(\partial Q^{lp}/\partial f_k\right) > 0$, $\tau = P_Q(\cdot) / \left[(1+\beta)P_F^f(\cdot)\right] > 0$ and $\mu = P_Q(\cdot)\left(\partial Q^{lp}/\partial f_j\right)\left(\partial q_i^{lp}/\partial f_j\right) > 0$. Results are straightforward.

A.9 Proof of Lemma 3

By applying the implicit function theorem on the system of first order conditions in (27)

$$\left.\frac{\partial f_j^{lp}}{\partial \beta}\right|_{\lambda_j^{lp}=0} = -\frac{\mu}{(1+\beta)P_F^f(\cdot) - \mu}\left[\frac{\partial q_j^{lp}}{\partial \beta} - \frac{P_F^f(\cdot)}{(1+\beta)P_F^f(\cdot) - \mu} q_j^{lp}\right] \tag{61}$$

$$\left.\frac{\partial f_i^{lp}}{\partial \beta}\right|_{\lambda_i^{lp}=0} = \left.\frac{\partial f_i^{lp}}{\partial \beta}\right|_{\lambda_i^{lp}\neq 0} = \left.\frac{\partial f_j^{lp}}{\partial \beta}\right|_{\lambda_j^{lp}\neq 0} = 0 \tag{62}$$

where $\Omega_4 = \left[(1+\beta)P_F^f(\cdot)\right]^2\left[\tau\frac{\partial Q^{lp}}{\partial f_j}\frac{\partial q_i^{lp}}{\partial f_j}\left(\frac{\partial q_j^{lp}}{\partial f_j} - 1\right) + 1\right] > 0$ and $\mu = P_Q(\cdot)\left(\partial Q^{lp}/\partial f_j\right)\left(\partial q_i^{lp}/\partial f_j\right) > 0$.

A.10 Proof of Proposition 6

Using the same method as in Proof of Prop.(4) we assume $q_k^{lp} = q_k^l + \varepsilon$ where $\varepsilon \simeq 0$. By Tylor expansion and FOCs

$$\Pi^k\left(q_i^l, q_j^l\right) \simeq \frac{1}{2}\Pi_{qq}^k\left(q_i^l, q_j^l\right)\left(q_k^l - q_k^{lp}\right)^2 \quad \text{and} \quad \Pi^k\left(q_i^{lp}, q_j^{lp}\right) \simeq \frac{1}{2}\Pi_{qq}^k\left(q_i^{lp}, q_j^{lp}\right)\left(q_k^l - q_k^{lp}\right)^2 \tag{63}$$

where

$$\Pi^j_{qq}\left(q_i,q_j\right) = -C_{qq}(q_j) \quad \text{and} \quad \Pi^i_{qq}\left(q_i,q_j\right) = 2P_Q(\cdot)\left(1-\tfrac{P_Q(\cdot)}{2P_Q(\cdot)-C_{qq}(\cdot)}\right) - C_{qq}(\cdot) \ . \quad (64)$$

Therefore profitability of forward market manipulation leads

$$\frac{1}{2}\left[C_{qq}(q_j^{lp}) - C_{qq}(q_j^{l})\right]\left(q_j^{lp}-q_j^{l}\right)^2 \leq 0 \quad (65)$$

for the follower and

$$\frac{1}{2}\left[2P_Q(Q^{lp})\tfrac{P_Q(Q^{lp})-C_{qq}(\cdot)}{2P_Q(Q^{lp})-C_{qq}(\cdot)} - C_{qq}(q_i^{lp}) - 2P_Q(Q^{l})\tfrac{P_Q(Q^{l})-C_{qq}(\cdot)}{2P_Q(Q^{l})-C_{qq}(\cdot)} + C_{qq}(q_i^{l})\right]\left(q_i^{lp}-q_i^{l}\right)^2 \geq 0 \quad (66)$$

for the leader. Recalling that spot market demand is downward sloping (i.e. $P_Q(\cdot) < 0$) and that costs are convex (i.e. $C_{qq}(\cdot) \geq 0$) we obtain

$$\Pi^k\left(q_i^{lp},q_j^{lp}\right) \geq \Pi^k\left(q_i^{l},q_j^{l}\right) \implies q_k^l > q_k^{lp} \ . \quad (67)$$

Finally using the formalizations in Lem.(2) we obtain that aggregate production turns out as an increasing of commitments and therefore

$$\Pi^k\left(q_k^{lp},q_{-k}^{lp}\right) \geq \Pi^k\left(q_k^{l},q_{-k}^{l}\right) \implies q_k^l > q_k^{lp} \implies Q^l > Q^{lp} \implies f_k^* > f_k^s \quad \forall k = \{i,j\} \ . \quad (68)$$

B Appendix B

B.1 Linear Cournot model

For analytical purposes and to strenghten comparisons between *ex-ante* (AV) and *ex-post* settings, we consider symmetric firms, linear costs structures and Cournot competition in the spot market (i.e. $\alpha = 0$). As a consequence, at equilibrium, duopolists will behave symmetrically *id est* produce equal amounts of power, commit to the same quantity of contracts and get the same profit. Results are presented in Tab.1.

	$P(Q)$	Q	F
Ex-ante	$(A+4c)/5$	$4(A-c)/5$	$2(A-c)/5$
Ex-post	$(A-B+2c)/2$	$(A+B-2c)/2$	$(3B-A-2c)/2$

Tab 1: Linear Counot model: ex-ante versus ex-post

Notice that both *ex-ante* and *ex-post* frameworks lead to partial contract coverage (i.e. $Q^* = (4/5)(A-c) > F^* = (2/5)(A-c)$ and $Q^s = (1/2)(A+B-2c) \geq F^s = (1/2)(3B-A-2c)$ where $A \neq B$). Furthermore recalling that $A, B, c > 0$, the conditions which must hold to have non-negative spot prices, quantities and equilibrium profits in both *ex-post* (actual) and *ex-ante* settings are

$$A - 2c = b_0 \leq B \leq b_1 = A \quad \text{and} \quad A \geq c \ . \tag{69}$$

Definition 4 *Forward market manipulation is profitable if it turns out to be profit enhancing (i.e. if it yields higher profits than the AV counterpart).*

Applying Def.(4), we find that notwithstanding the independence of *ex-post* (actual) equilibria from β, manipulation of forward markets is profitable if

$$(8c - 3A)/5 = b_2 \leq B \leq b_3 = (3A + 2c)/5 \tag{70}$$

id est where the upsurge in power prices balances the decrease in the quantity produced and sold by the industry. Notice that within the relevant interval (i.e. $\forall B \in [b_2; b_3]$), consistently with Prop.(4) the following verifies

$$Q^* > Q^s, \quad q^* > q^s \quad \text{and} \quad F^* > F^s \ . \tag{71}$$

Simulating linear Cournot models. To further investigates the outcomes in *ex-ante* and *ex-post* frameworks, a simulation has been carried out. In the proceeding we provide further support to both pro and anti-competitive natures of forward commitments and confirm that parametrization is crucial. As usual the superscript '*' is used for *ex-ante* settings while '^{s}' stands for the *ex-post* (actual) counterpart.

First, assume the technological structure of the industry and demand functions (intercepts) are as follows:

$$c \in [0.05; 1.00], \quad A = 1.10c \quad \text{and} \quad B = 1.08c \tag{72}$$

then manipulation of forward markets is profit detrimental (see Fig.(B.1)). *Ex-ante* models await higher prices and lower commitments than *ex-post.*

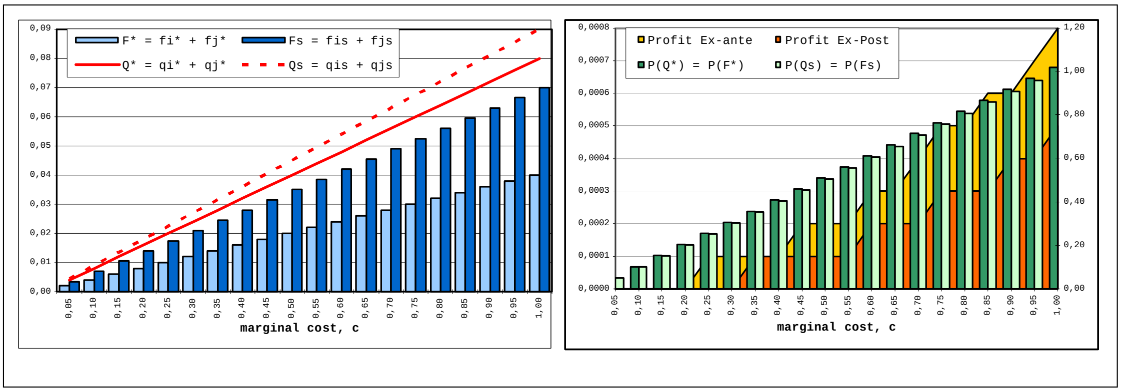

Fig.(B.1). (Pro-competitive). Ex-ante versus ex-post: selling contracts.

Second, assume the technological structure of the industry and demand functions (intercepts) are similar to those in (72) except for B and let

$$c \in [0.05; 1.00]\,, \quad A = 1.10c \quad \text{and} \quad B = 1.05c\ , \tag{73}$$

then manipulation of forward markets is profit enhancing (see Fig.(B.2)). *Ex-ante* models await lower prices and higher commitments than *ex-post.*

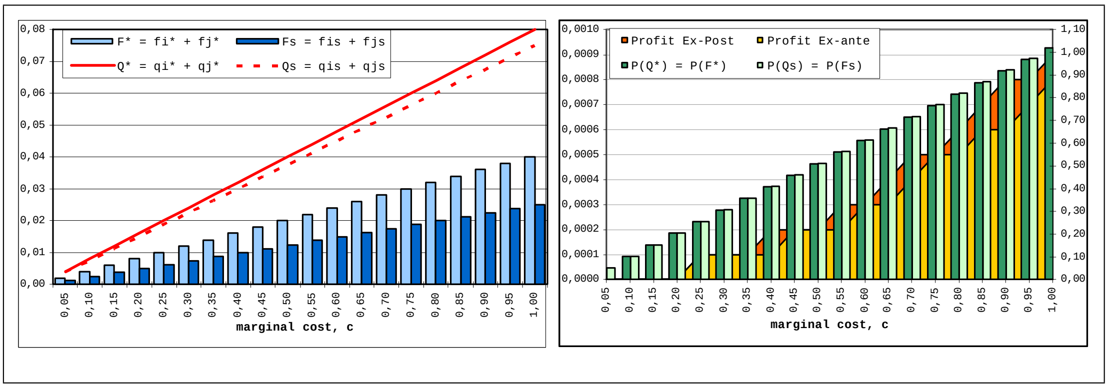

Fig.(B.2). (Anti-competitive). Ex-ante versus ex-post: selling contracts.

Third, assume the technological structure of the industry and demand functions (intercepts) are

$$c \in [0.05; 1.00]\,, \quad A = 1.10c \quad \text{and} \quad B = c\ . \tag{74}$$

Notwithstanding the small difference with parametrization in (73), manipulation of forward markets lead buying attitudes so the anti-competitive effect is stronger (Fig.(B.3)).

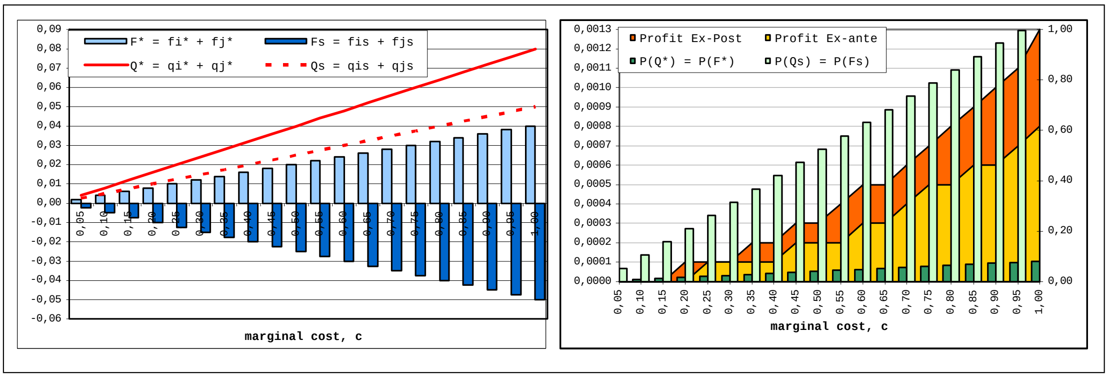

Fig.(B.3). (Anti-competitive). *Ex-ante* versus *ex-post*: buying contracts.

Concluding remark on Lagrange multipliers. Consider the *ex-post* setting. When the constraint is not binding (i.e. $\lambda = 0$), contracts turn out as a function of the technological structure of the industry (i.e. c) as well as of the degree of market power (i.e. β) in the contract market (cfr. Lem.(3)):

$$f_i^s|_{\lambda=0} = \frac{9B-2A-c}{(9\beta+23)}. \tag{75}$$

The difference between contracts with non binding constraint and actual contracts with binding constraint depends on β.

Simple calculations show that in the range $b_0 < B < b_1$:

- with $\beta = 0$, (Cournot conjectures in the contract market), positive profit in the contract market (with forward price larger than spot price) allows to increase contracts, so that unconstrained contracts are always smaller than actual ones. The contract cover being larger, spot price that realizes when the constraint is binding decreases with respect to the case with non-binding constraint.

- with $\beta = -1$, (Bertrand conjectures in the contract market), it exists a threshold $(B < (5A + 8c)/17)$ such that actual contracts are always smaller than potential ones, the forward price is larger than spot price, and the latter is larger with respect to the spot price that realizes with binding constraint.

B.2 Linear Stackelberg model

To solve the Stakelberg model, we allow for cost differences between the two players. In particular we let $c_i = c > c_j = 0$ where, as usual, i denotes the leader and j stands for the follower. As a benchmark, let us remind the standard results of Stackelberg competition:

$$P(Q) = \tfrac{A+c}{4} \quad \text{and} \quad Q = \tfrac{3A-2c}{4} \ . \tag{76}$$

Table 2 presents results for Stackelberg leadership with forward commitments and *ex-ante* as well as *ex-post* perfect arbitrage.

	$P(Q)$	Q	F
Ex-ante	$(A+2c)/6$	$(5A-2c)/6$	$(A+2c)/3$
Ex-post	$(A-B+2c)/3$	$(2A+B-2c)/3$	$(3B-A-2c)/2$

Tab 2: Linear Stackelberg model: ex-ante versus ex-post

Recall that when the arbitrage condition holds *ex-ante*, at the equilibrium the leader does not contract; therefore $F^l = f^l_j$. Compared to the standard case in (76) when perfect arbitrage holds *ex-ante*, forward markets alleviate market distortions and lead lower spot price.

The conditions which must hold to have non-negative spot prices and quantities in both *ex-post* and *ex-ante* settings are

$$2(A-c) = a_0 < B < a_2 = (A+2c) \quad \text{and} \quad A > \tfrac{2}{5}c \ . \tag{77}$$

Along the intervals in (77) there is partial contract coverage (i.e. $Q^l < F^l = f^l_j$ and $Q^{lp} < F^{lp}$). Furthermore as $F^{lp} < F^l$, *id est ex-ante* commitments outweight *ex-post*'s, manipulation of forward market is always profitable.

Simulating linear Stackelberg models. To further investigates the outcomes in *ex-ante* and *ex-post* frameworks, a simulation has been carried out. As usual the superscript '^{l}' is used for *ex-ante* settings while '^{lp}' stands for the *ex-post* counterpart.

First, assume the technological structure of the industry and demand functions (intercepts) are as follows:

$$c \in [0.05; 1.00], \quad A = 5.00c \quad \text{and} \quad B = 3.00c \tag{78}$$

then the awaited pattern results (see Fig.(B.4)). Contracts alter the profit distribution between the leader and the follower. In *ex-ante* settings the follower contracts to counterbalence spot market dominance while in *ex-post* frameworks both firms commit forward. Although both agent benefit of forward market manipulation, we expect the "small" firm to collect major advantages.

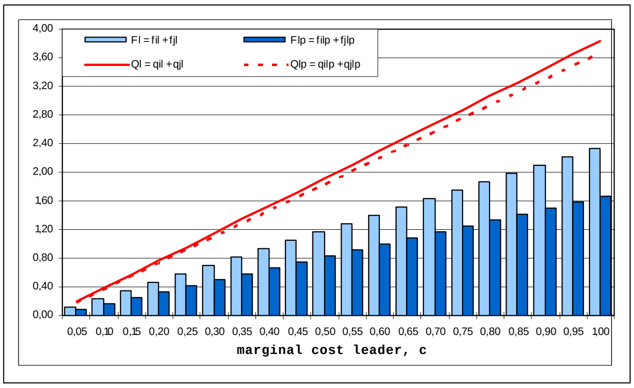

Fig.(B.4). Stackelberg *ex-ante* versus *ex-post*: selling contracts.

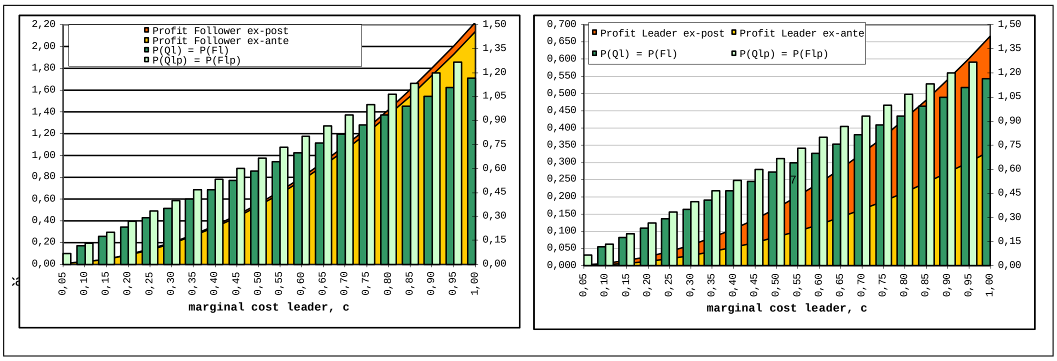

Fig.B.5. Stackelberg *ex-ante* versus *ex-post*: selling contracts.

Second, assume the technological structure of the industry and demand functions (intercepts) are as follows:

$$c \in [0.05; 1.00], \quad A = 3.70c \quad \text{and} \quad B = 0.30c \tag{79}$$

then the awaited pattern results (see Fig.(B.6)-(B.7)).

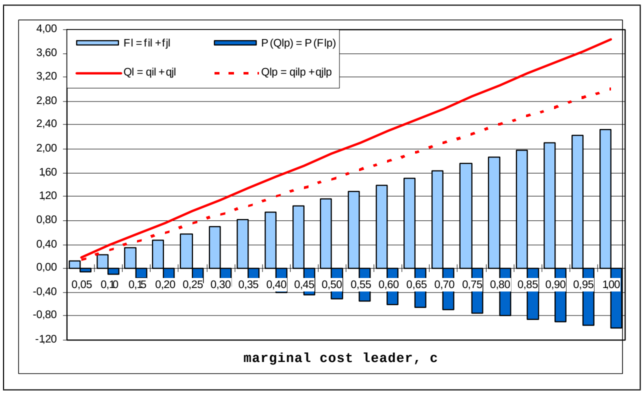

Fig.(B.6). Stackelberg *ex-ante* versus *ex-post*: buying contracts.

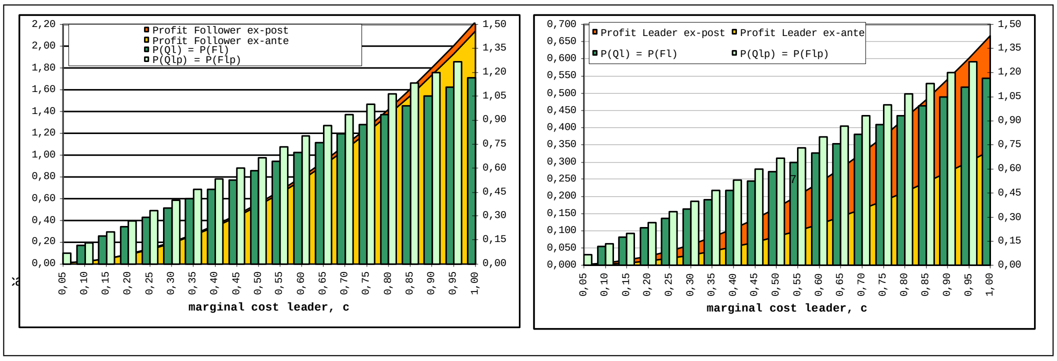

Fig.(B.7). Stackelberg *ex-ante* versus *ex-post*: buying contracts.